Fourierova transformace distribucí

(1): $f \in L^1(\mathbb{R}^N)$
$\mathcal{F}(f) = \int_{\mathbb{R}^N} f(x) \exp\{-2\pi \mathrm{i}(x,\xi)\}\,\mathrm{d}x$

(2): $\mathbb{A} \in \mathbb{R}^{N\times N}$, poz. definitní, symetrická
$\mathcal{F}(\exp\{-(\mathbb{A}x,x)\}) = \frac{(\sqrt{\pi})^N}{\sqrt{|\det \mathbb{A}|}} \exp\{-\pi^2(\mathbb{A}^{-1}\xi,\xi)\}$

(3): $\delta \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(\delta) = 1$

(4): $T_1 \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(T_1) = \delta$

(5): $T_{x^n} \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(T_{x^n}) = \frac{1}{(-2\pi\mathrm{i})^n} D^n\delta$

(6): $D^n\delta \in \mathcal{S}'(\mathbb{R})$, $n \in \mathbb{N}$
$\mathcal{F}(D^n\delta) = (2\pi\mathrm{i})^n \xi^n$

(7): $b \in \mathbb{C}$
$\mathcal{F}\left(T_{\exp(2\pi\mathrm{i}bx)}\right) = \delta_b$

(8): $b \in \mathbb{C}$
$\mathcal{F}\left(T_{\sin(2\pi bx)}\right) = \frac{1}{2\mathrm{i}}(\delta_b - \delta_{-b})$

(9): $b \in \mathbb{C}$
$\mathcal{F}\left(T_{\cos(2\pi bx)}\right) = \frac{1}{2}(\delta_b + \delta_{-b})$

(10): $b \in \mathbb{C}$
$\mathcal{F}\left(T_{\sinh(2\pi bx)}\right) = \frac{1}{2}(\delta_{-\mathrm{i}b} - \delta_{\mathrm{i}b})$

(11): $b \in \mathbb{C}$
$\mathcal{F}\left(T_{\cosh(2\pi bx)}\right) = \frac{1}{2}(\delta_{-\mathrm{i}b} + \delta_{\mathrm{i}b})$

(12): $H_{x_+^\lambda} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$
$\mathcal{F}\left(\frac{H_{x_+^\lambda}}{\Gamma(\lambda+1)}\right) = \mathrm{e}^{-\mathrm{i}(\lambda+1)\frac{\pi}{2}}(2\pi)^{-\lambda-1} H_{(\xi-\mathrm{i}0)^{-\lambda-1}}$

(13): $x_+^n \in \mathcal{S}'(\mathbb{R})$, $n \in \mathbb{N}$
$\mathcal{F}\left(H_{x_+^n}\right) = (2\pi\mathrm{i})^{-n-1} n! H_{\xi^{-n-1}} + \frac{1}{2}(2\pi\mathrm{i})^{-n}(-1)^{-n} D^n\delta$

(14): $T_H \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(T_H) = \mathcal{F}\left(H_{x_+^0}\right) = \frac{1}{2\pi\mathrm{i}} T_{\text{v.p.}\,\xi^{-1}} + \frac{1}{2}\delta$

(15): $H_{x_-^\lambda} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$
$\mathcal{F}\left(\frac{H_{x_-^\lambda}}{\Gamma(\lambda+1)}\right) = \mathrm{e}^{\mathrm{i}(\lambda+1)\frac{\pi}{2}}(2\pi)^{-\lambda-1} H_{(\xi+\mathrm{i}0)^{-\lambda-1}}$

(16): $H_{|x|^\lambda} = H_{x_+^\lambda} + H_{x_-^\lambda} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$, $\lambda \neq -n$, $n \in \mathbb{N}$
$\mathcal{F}\left(H_{|x|^\lambda}\right) = -2\Gamma(\lambda+1)(2\pi)^{-\lambda-1} \sin\left(\frac{\pi}{2}\lambda\right) H_{|\xi|^{-\lambda-1}}$

(17): $H_{|x|^\lambda \operatorname{sign} x} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$; $\mathcal{F}\left(H_{|x|^\lambda \operatorname{sign} x}\right)$
$= -2\mathrm{i}(2\pi)^{-\lambda-1}\Gamma(\lambda+1) \cos\left(\frac{\pi}{2}\lambda\right) H_{|\xi|^{-\lambda-1}\operatorname{sign}\xi}$

(18): $H_{x^{-m}} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$, $m \in \mathbb{N}$; $\mathcal{F}(H_{x^{-m}})$
$= \begin{cases} (-1)^{\frac{m+1}{2}} \mathrm{i}\pi \frac{(2\pi)^{m-1}}{(m-1)!} H_{|\xi|^{m-1}\operatorname{sign}\xi} & m \text{ liché} \\ (-1)^{\frac{m}{2}} \frac{\pi(2\pi)^{m-1}}{(m-1)!} H_{|\xi|^{m-1}} & m \text{ sudé} \end{cases}$

(19): $T_{\text{v.p.}\,x^{-1}} \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}\left(T_{\text{v.p.}\,x^{-1}}\right) = -\mathrm{i}\pi T_{\operatorname{sign}\xi}$

(20): $H_{x^{-2}} \in \mathcal{S}'(\mathbb{R})$,
$\mathcal{F}(H_{x^{-2}}) = -T_{|\xi|} 2\pi^2$

(21): $H_{(x+\mathrm{i}0)^\lambda} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$
$\mathcal{F}\left(H_{(x+\mathrm{i}0)^\lambda}\right) = \frac{H_{\xi_+^{-\lambda-1}}}{\Gamma(-\lambda)} \exp\{\mathrm{i}\lambda\frac{\pi}{2}\}(2\pi)^{-\lambda}$

(22): $H_{(x-\mathrm{i}0)^\lambda} \in \mathcal{S}'(\mathbb{R})$, $\lambda \in \mathbb{C}$
$\mathcal{F}\left(H_{(x-\mathrm{i}0)^\lambda}\right) = \frac{H_{\xi_-^{-\lambda-1}}}{\Gamma(-\lambda)} \exp\{-\mathrm{i}\lambda\frac{\pi}{2}\}(2\pi)^{-\lambda}$

(23): $r = |x|, x \in \mathbb{R}^N$, $\lambda \in \mathbb{C}$, $\rho = |\xi|, \xi \in \mathbb{R}^N$
$\mathcal{F}\left(\frac{H_{r^\lambda}}{\Gamma\left(\frac{\lambda+N}{2}\right)}\right) = \frac{H_{\rho^{-\lambda-N}}}{\Gamma(-\lambda/2)\pi^{\lambda+N/2}}$