# Maxwellove rovnice

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

# Transformácie polí

$$F'_{\mu\nu} = \Lambda^{\mu'}_{\mu} F_{\mu'\nu'} \Lambda^{\nu'}_{\nu}$$

$$E'_{\parallel} = E_{\parallel} \qquad E'_{\perp} = \left(E_{\perp} + \frac{\vec{v}}{c}\times c\vec{B}\right)\gamma$$

$$B'_{\parallel} = B_{\parallel} \qquad B'_{\perp} = \left(cB_{\perp} - \frac{\vec{v}}{c}\times\vec{E}\right)\gamma$$

$$\frac{\phi'}{c} = \gamma\left(\frac{\phi}{c} - \frac{v}{c}A_{\parallel}\right)$$

$$A'_{\parallel} = \gamma\left(A_{\parallel} - \frac{v}{c}\frac{\phi}{c}\right)$$

# Biot-Savart

$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{j}'}{|\vec{x}-\vec{x}'|}dV$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\vec{j}(\vec{x}')\times\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}dV$$

$$\vec{B} = \frac{\mu_0}{4\pi}I\int d\vec{s}\times\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}$$

# Integrálny Ampér

$$\oint_{\Gamma}\vec{B}\cdot d\vec{l} = \mu_0 I$$

# Dipól

$$\vec{m} = \frac{1}{2}\int\vec{r}\times\vec{j}\,dV$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m\times\vec{r}}{|r|^3}$$

$$\vec{F} = \vec{m}\cdot\nabla\vec{B}$$

$$\vec{M} = \vec{m}\times\vec{B}$$

# Magnetická sila

$$\vec{F} = \int\vec{j}\times\vec{B}\,dV$$

# Indukčnosť

$$L = \frac{\mu_0}{4\pi}\oint_{\Gamma_l}\oint_{\Gamma_k}\frac{d\vec{s}_k\cdot d\vec{s}_l}{|\vec{x}_k-\vec{x}_l|}$$

$U = \frac{1}{2}LI^2$ ~> 1 slučka

$U = \frac{1}{2}I_k L_{kl} I_l$ ~> viacej slučiek

$$\Psi_k = L_{kl}I_l$$

# Magnetický tok

$$\Psi = \int\vec{B}\cdot d\vec{S}$$

$$-\frac{d\Psi}{dt} = \mathcal{E}$$

# Energia

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$u_B = \frac{1}{2}\varepsilon_0 B^2$$

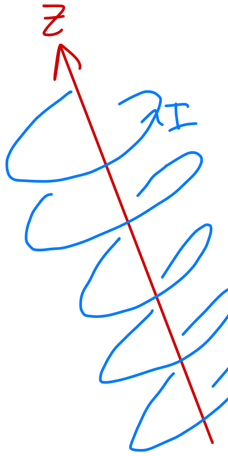
$$\vec{S} = \varepsilon_0 c^2\vec{E}\times\vec{B}$$

$$\vec{g} = \frac{1}{c^2}\vec{S}$$

$$b = \nabla\times a$$

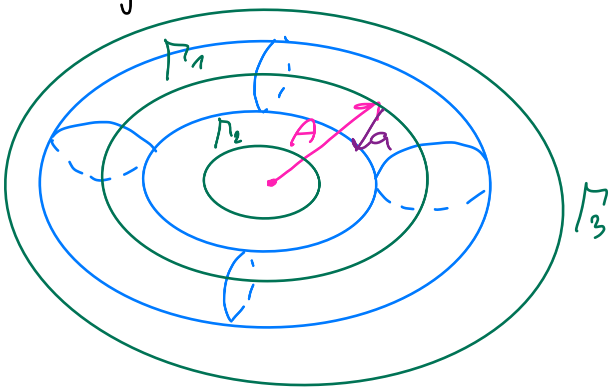
$$b^1 = \frac{1}{h_1h_2h_3}\left((h_3a^3)_{,2} - (h_2a^2)_{,3}\right)$$

# 10.2 Pole nekonečného solenoidu




$n = \frac{N}{L}$ ... hust. vinutia

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$




$$\int_{\Gamma_2} \vec{B} \cdot d\vec{l} = 0 \Rightarrow B = 0 \qquad 0 \leq d \leq A - a$$

$$\int_{\Gamma_3} \vec{B} \cdot d\vec{l} = \mu_0 (NI - NI) \Rightarrow B = 0 \qquad d \geq A + a$$

$$\int_{\Gamma_1} \vec{B} \cdot d\vec{l} = \mu_0 NI = 2\pi r B \qquad A - a < r < A + a \qquad |\overbrace{d - A}^{r}| < a$$

$$\Rightarrow B = \frac{\mu_0}{\cancel{2\pi}} \frac{NI}{d} = \frac{\mu_0}{\cancel{2\pi}} \frac{n \cancel{2\pi}(A-a)}{d} I \qquad \Updownarrow \quad A - a < d < A + a$$

$$B = \mu_0 n \frac{A - a}{A + r} I \xrightarrow{A \to \infty} B = \mu_0 n I \quad \text{vnú}$$

$$\Rightarrow \boxed{B = \begin{cases} \mu_0 n I \vec{e}_z & r < a \\ 0 & r > a \end{cases}}$$

→ na povrchu:

$$[\vec{n} \times \vec{B}] = \mu_0 \iota$$

$$\vec{n} = -\vec{e}_r \Rightarrow -\mu_0 n I \underbrace{\vec{e}_r \times \vec{e}_z}_{-\vec{e}_\varphi} = \mu_0 \vec{\iota} \Rightarrow \vec{\iota} = n I \vec{e}_\varphi = \iota \vec{e}_\varphi$$

$$\Rightarrow \boxed{\vec{B} = \mu_0 \iota \vec{e}_z}$$

→ Pre dva solenoidy s polomermi $\overset{\iota_2}{a_2} < \overset{\iota_1}{a_1}$ bude pole:

$$\boxed{\vec{B} = \begin{cases} \mu_0 (\iota_2 + \iota_1) & r < a_2 \\ \mu_0 \iota_1 & a_2 < r < a_1 \\ 0 & a_1 < r \end{cases}}$$

## 10.3 Vektorový potenciál homo. mag. pole

$$\vec{B} = B\vec{e}_z \qquad B_i = \varepsilon_{ijk}\partial_j A_k$$

$$\vec{B} = \nabla\times\vec{A} \rightsquigarrow B = \partial_x A_y - \partial_y A_x = B \;,\quad A_z = 0$$

$$A_x = By \qquad A'_x = -By$$
$$A_y = 2Bx \qquad A'_y = 0$$

$$\vec{A} = B(y, 2x, 0) \qquad \vec{A}' = B(-y, 0, 0)$$

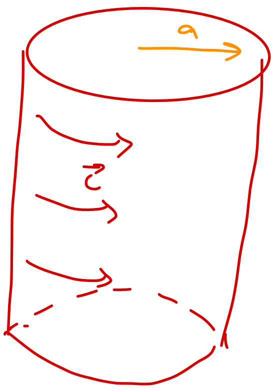
$$\nabla\psi = B(-2y, -2x, 0) \Rightarrow \psi = -2Bxy$$

$$\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B} \Rightarrow A_i = -\frac{1}{2}\varepsilon_{ijk}x_j B_k$$

$$\nabla\times\vec{A} \to \varepsilon_{ijk}\partial_j A_k = \varepsilon_{ijk}\partial_j\, -\frac{1}{2}\varepsilon_{klm}x_l B_m = -\frac{1}{2}\underset{kji}{\varepsilon_{ijk}}\varepsilon_{klm}\delta_{lj}B_m =$$
$$= \frac{1}{2}\left(\delta_j^l\delta_i^m - \delta_j^m\delta_i^l\right)\delta_{lj}B_m = \frac{1}{2}\left(\delta_j^j\delta_i^m - \delta_i^m\right)B_m =$$
$$= \frac{1}{2}(3-1)B_i = B_i$$

$$\psi = \int_S \vec{B}\cdot d\vec{S} = \int_S \nabla\times\vec{A}\cdot d\vec{S} = \oint_{\partial S}\vec{A}\cdot d\vec{l}$$

## 10.4 Vektorový potenciál nekonečného solenoidu

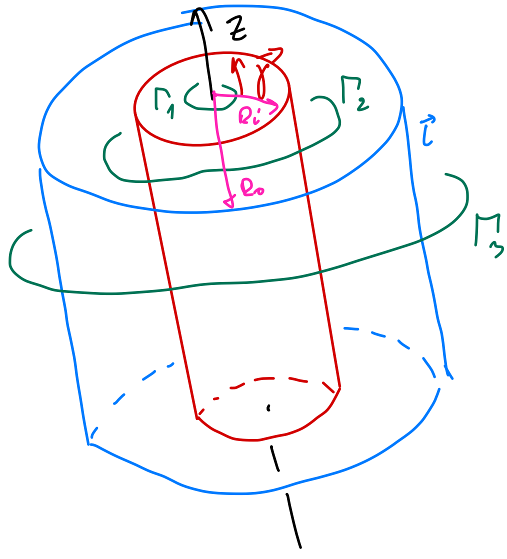



$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{j}(\vec{r}')}{|\vec{x}-\vec{x}'|}dV =$$
$$= \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\int_0^{2\pi}\frac{ca}{|\vec{x}-\vec{x}'|}d\varphi\, dz$$

$$\left[\, |\vec{x}-\vec{x}'|^2 = (r\cos\varphi - a\cos\varphi')^2 + (r\sin\varphi - a\sin\varphi')^2 + (z-z')^2 = \right.$$
$$\left. = r^2 + a^2 - 2ra\cos(\varphi-\varphi') + (z-z')^2 \,\right]$$

$$= \frac{\mu_0 ac}{4\pi}\int_0^{2\pi}\int_{-\infty}^{\infty}\frac{1}{\left(r^2+a^2-2ra\cos(\varphi-\varphi')+z'^2\right)^{\frac{1}{2}}}dz'\,d\varphi$$

# 10.5 Magnetické pole koaxiálneho vodiče




$$\vec{j} = j\vec{e}_z \qquad \vec{i} = i\vec{e}_z$$

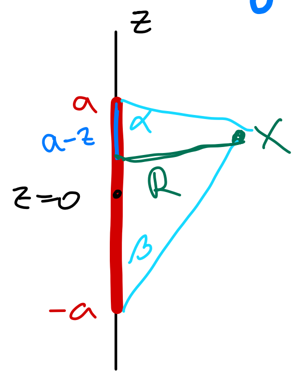
$$\oint_{\Gamma_3} \vec{B}\cdot d\vec{l} = \mu_0\left(j\cdot S + i\,l\right) = 2\pi r B$$

$$\oint_{\Gamma_2} \vec{B}\cdot d\vec{l} = \mu_0\, j S = 2\pi r B$$

$$\oint_{\Gamma_3} \vec{B}\cdot d\vec{l} = \mu_0\, j \pi r^2 = 2\pi r B$$

$$\Rightarrow B_\varphi = \begin{cases} \frac{\mu_0}{2\pi}\frac{1}{r}\left(j\pi R_i^2 + 2\pi R_o i\right) = \frac{\mu_0}{2r}\left(j R_i^2 + 2R_o i\right) & r > R_i \\ \frac{\mu_0}{2\pi}\frac{1}{r}\left(j\pi R_i^2\right) = \frac{\mu_0}{2r} j R_i^2 & R_o < r < R_i \\ \frac{\mu_0}{2} j r & R_o > r \end{cases}$$

# 10.6 Magnetické pole v přímem úseku vodiče




$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}' \times (\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

$$\left.\begin{aligned} \vec{x}' &= (0, 0, z') \\ \vec{x} &= (r\cos\varphi, r\sin\varphi, z) \end{aligned}\right\} \vec{x}-\vec{x}' = \vec{e}_x r\cos\varphi + \vec{e}_y r\sin\varphi + (z-z')\vec{e}_z$$

$$d\vec{l}' = (0, 0, dz') = \vec{e}_z\, dz'$$

$$d\vec{l}' \times \vec{r}' = r\cos\varphi\, dz'\, \overbrace{\vec{e}_z \times \vec{e}_x}^{\vec{e}_y} + r\sin\varphi\, dz'\, \overbrace{\vec{e}_z \times \vec{e}_y}^{-\vec{e}_x}$$

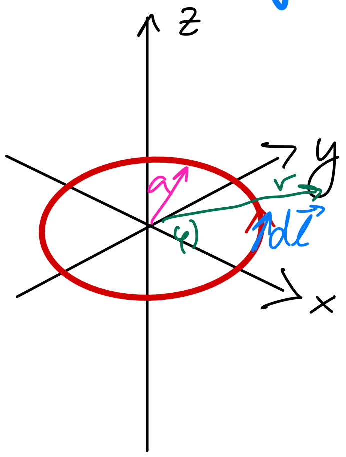
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{r\cos\varphi\, dz'\,\vec{e}_y - r\sin\varphi\, dz'\,\vec{e}_x}{\left(r^2 + (z-z')^2\right)^{\frac{3}{2}}} =$$

$$= \mu_0 \frac{I}{4\pi}\left(r\cos\varphi\,\vec{e}_y - r\sin\varphi\,\vec{e}_x\right)\int_{-a-z}^{a-z} \frac{dz'}{\left(r^2 + z'^2\right)^{\frac{3}{2}}} =$$

$$= \mu_0 \frac{I}{4\pi}\left(r\cos\varphi\,\vec{e}_y - r\sin\varphi\,\vec{e}_x\right)\left[\frac{z'/R^2}{\left(R^2+z'^2\right)^{\frac{1}{2}}}\right]_{R\,\mathrm{ctg}\,\alpha}^{R\,\mathrm{ctg}\,\beta} =$$

$$= \mu_0 \frac{I}{4\pi}\underbrace{\left(\cos\varphi\,\vec{e}_y - \sin\varphi\,\vec{e}_x\right)}_{\vec{e}_\varphi}\left[\frac{\mathrm{ctg}\,\beta}{R(1+\mathrm{ctg}^2\beta)^{\frac{1}{2}}} - \frac{\mathrm{ctg}\,\alpha}{R\,\underbrace{(1+\mathrm{ctg}^2\alpha)}_{\frac{\sin^2+\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha}}^{\frac{1}{2}}}\right] = \boxed{\mu_0\frac{I}{4\pi R}\left(\cos\alpha - \cos\beta\right)\vec{e}_\varphi}$$

# 10.7 Magnetické pole proudu v krukové smyčce




$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}$$

$$\vec{x}' = (a\cos\varphi', a\sin\varphi', 0)$$

$\Big\}$ dotyčnica

$$d\vec{l}' = (-a\sin\varphi', a\cos\varphi', 0)\, d\varphi'$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{(-a\sin\varphi', a\cos\varphi', 0) \times (r\cos\varphi - a\cos\varphi', r\sin\varphi - a\sin\varphi', z)\, d\varphi'}{|\vec{x} - \vec{x}'|^3}$$

$$|\vec{x} - \vec{x}'|^2 = (r\cos\varphi - a\cos\varphi')^2 + (r\sin\varphi - a\sin\varphi')^2 + (z - z')^2 =$$

$$= r^2 + a^2 - 2ra\cos\varphi\cos\varphi' - 2ra\sin\varphi\sin\varphi' + (z - z')^2 =$$

$$= r^2 + a^2 - 2ra\cos(\varphi - \varphi') + z^2$$

$$d\vec{l} \times (\vec{x} - \vec{x}') = \big(a\cos\varphi' z,\ a\sin\varphi' z,$$
$$-a\sin\varphi'(r\sin\varphi - a\sin\varphi') - a\cos\varphi'(r\cos\varphi - a\cos\varphi')\big) =$$

$$= \left(a\cos\varphi' z, -a\sin\varphi' z, -ar\cos(\varphi - \varphi') + a^2\right)$$

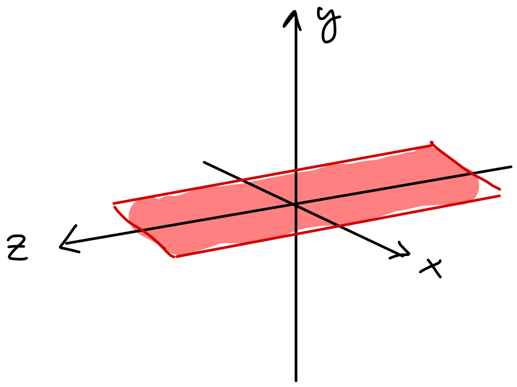
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{\left(a\cos\varphi' z,\ a\sin\varphi' z, -ar\cos(\varphi - \varphi') + a^2\right)}{\left(r^2 + a^2 - 2ra\cos(\varphi - \varphi') + z^2\right)^{\frac{3}{2}}}\, d\varphi'$$

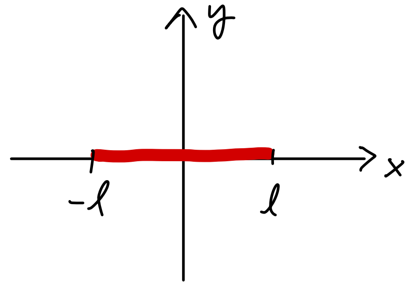
- na ose: $r = 0$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{(az\cos\varphi', az\sin\varphi' z, a^2)}{(a^2 + z^2)^{\frac{3}{2}}}\, d\varphi =$$

$$= \frac{\mu_0 I}{2} \frac{(0, 0, a^2)}{(a^2 + z^2)^{\frac{3}{2}}}$$

# 10.9 Magnetické pole proudu tekoucího plošným vodičem




$\vec{i} = i\,(0,0,1)$

x z y
y z x

$$\vec{B} = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty}\int_{-l}^{l} \frac{(0,0,1)\times(x-x', y, z-z')}{\left((x-x')^2+y^2+(z-z')^2\right)^{\frac{3}{2}}}\, dx'dz' =$$

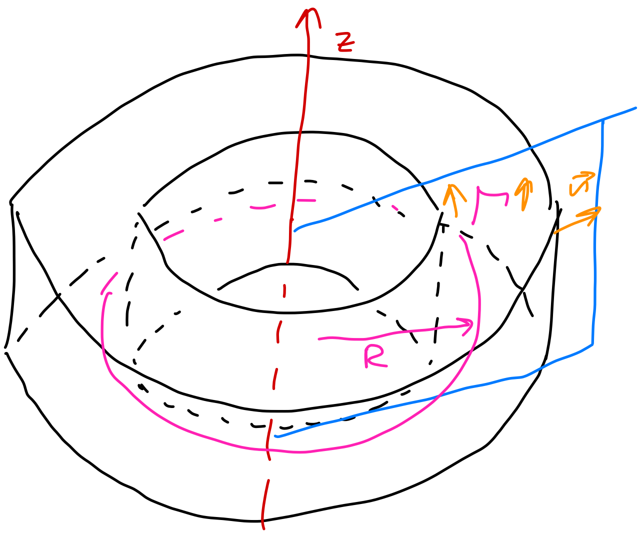
$$= \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty}\int_{-l}^{l} \frac{\vec{e}_y(x-x') - \vec{e}_x y}{\left((x-x')^2+y^2+(z-z')^2\right)^{\frac{3}{2}}}\, dx'dz' =$$

$$= \frac{\mu_0 i}{4\pi} \int_{-l}^{l} \vec{e}_y(x-x') - \vec{e}_x y \underbrace{\int_{-\infty}^{\infty} \frac{1}{\left((x-x')^2+y^2+(z-z')^2\right)^{\frac{3}{2}}}\, dz'}_{\left[\frac{z'/\left[(x-x')^2+y^2\right]}{\left((x-x')^2+y^2+z'^2\right)^{\frac{1}{2}}}\right]_{-\infty}^{\infty} = \frac{2}{(x-x')^2+y^2}}\, dx'$$

$$= \frac{\mu_0 i}{2\pi} \int_{-l}^{l} \frac{\vec{e}_y(x-x') - \vec{e}_x y}{(x-x')^2+y^2}\, dx' =$$

$$= \frac{\mu_0 i}{2\pi} \left( \vec{e}_y \frac{1}{2} \ln\left(\frac{(x+l)^2+y^2}{(x-l)^2+y^2}\right) - \vec{e}_x \left[\operatorname{arctg}\left(\frac{x-l}{y}\right) - \operatorname{arctg}\left(\frac{x+l}{y}\right)\right]\right)$$

# 10.10 Magnetické pole solenoidu




$$\vec B = B\vec e_\varphi$$

$$\oint_\Gamma \vec B\cdot d\vec l = I\mu_0$$

$$2\pi R\, B = I\mu_0 \Rightarrow B = \frac{I\mu_0}{2\pi R}$$

$$\oint_{\Gamma'} \vec B_{\text{vně}}\cdot d\vec l = 0 \Rightarrow \vec B_{\text{vně}} = 0$$

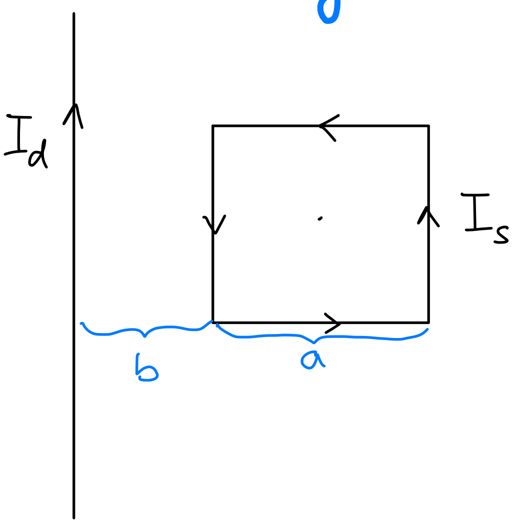
$$\vec B = \frac{I\mu_0}{2\pi R}\vec e_\varphi \qquad \partial_\varphi B = 0 \qquad \nabla\times\vec B \Rightarrow \frac{1}{R}\partial_R(RB) = 0 \qquad -\partial_z B = 0$$

$$\vec n = n_R\vec e_R + n_z\vec e_z$$

$$\vec n\times\vec B = \frac{I\mu_0}{2\pi R}\vec n\times\vec e_\varphi = \mu_0\vec\iota \Rightarrow \vec\iota = \frac{I}{2\pi R}\vec n\times\vec e_\varphi = \iota\,\vec n\times\vec e_\varphi = \frac{I}{2\pi R}\left(n_R\vec e_z - n_z\vec e_R\right)$$

$$\vec n\cdot\vec B = (\dots)\,\vec e_R\cdot\vec e_\varphi + (\dots)\,\vec e_z\cdot\vec e_\varphi = 0$$

# 10.11 Přímý vodič a čtvercová proudová smyčka

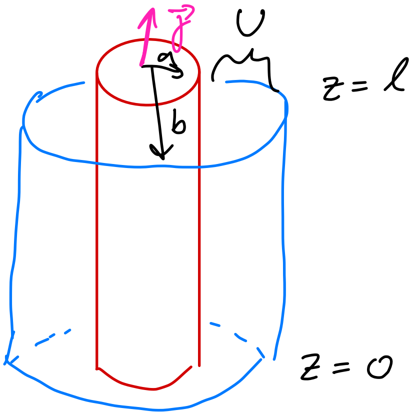



$$\vec B_d = \frac{\mu_0 I_d}{2\pi}\frac{1}{r}\vec e_\varphi = B\vec e_\varphi$$

$$\vec F = I_s\int d\vec s\times\vec B = \underbrace{\cancel{I_s\int_b^{b+a} d\vec r\times\vec B + I_s\int_{b+a}^{b} d\vec r\times\vec B}}_{\text{ze symetrie } 0} + I_s\int_0^a B(b+a)\,\vec e_z\times\vec e_\varphi\, dz + I_s\int_a^0 B(b)\,\vec e_z\times\vec e_\varphi\, dz =$$

$$= -I_s\vec e_r\,\frac{\mu_0 I_d\, a}{\pi}\left(\frac{1}{b+a} - \frac{1}{b}\right)$$

# 10.13 Koaxiální vodič zapojený do obvodu




$$\vec{E}=0 \qquad r>b$$

$$\gamma\vec{E}=\vec{j}=j\vec{e}_z \qquad r<a$$

0 symetrie $\qquad \phi=\tilde{\phi}z$

$$a<r<b: \quad \frac{1}{r}\partial_r(r\partial_r\phi)+\cancel{\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}}+\cancel{\frac{\partial^2\phi}{\partial z^2}}=0$$

$$\Rightarrow r\partial_r\phi=c \Rightarrow \phi=c\ln\frac{r}{r_0} \Rightarrow \phi=cz\ln\frac{r}{r_0}$$

$$\phi(z=\ell, r=b)=c\ell\ln\frac{b}{r_0}=0 \Rightarrow r_0=b$$

$$\phi(z=\ell, r=a)=c\ell\ln\frac{a}{b}=U \Rightarrow c=\frac{U}{\ln\frac{a}{b}\ell} \Rightarrow \phi=\frac{Uz}{\ell}\frac{\ln\frac{r}{b}}{\ln\frac{a}{b}}$$

$$\vec{E}=-\nabla\phi=-\frac{U}{\ell}\frac{\ln\frac{r}{a}}{\ln\frac{b}{a}}\vec{e}_z-\frac{Uz}{\ell r\ln\frac{b}{a}}\vec{e}_r \qquad a<r<b$$

$$[\vec{n}\times\vec{E}]=0 \Rightarrow \frac{j}{\gamma}-\frac{U}{\ell}=0 \Rightarrow j=\frac{U\gamma}{\ell} \Rightarrow I=\frac{U\gamma}{\ell}\pi a^2$$

$$\Rightarrow R=\frac{\ell}{\gamma\pi a^2}$$

## 11.1 Magnetické pole dipólu

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3} \Rightarrow A_i = \frac{\mu_0}{4\pi}\frac{1}{r^3}\varepsilon_{ijk}\, m_j\, x_k \qquad \frac{1}{(\dots)^{\frac{3}{2}}}$$

$$B_i = \varepsilon_{ijk}\,\partial_j A_k = \frac{\mu_0}{4\pi} m_\ell\, \varepsilon_{ijk}\, \varepsilon_{k\ell m}\, \partial_j \frac{x_m}{r^3} =$$

$$= \frac{\mu_0}{4\pi} m_\ell \left(\delta_i^\ell \delta_j^m - \delta_i^m \delta_j^\ell\right) \frac{\delta_j^m r^3 - x_m 3r\, x_j}{r^6} =$$

$$= \frac{\mu_0}{4\pi}\left(\delta_j^m m_i - \delta_i^m m_j\right)\frac{\delta_j^m r^2 - 3x_m x_j}{r^5} =$$

$$= \frac{\mu_0}{4\pi}\left(\frac{3r^2 m_i - \cancel{3r^2 m_i}\xrightarrow{} 0}{r^4} - \frac{m_i r^2 - 3x_i x_j m_j}{r^5}\right) =$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(\frac{3x_i x_j m_j}{r^2} - m_i\right) \Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(\frac{3\vec{r}(\vec{r}\cdot\vec{m})}{r^2} - \vec{m}\right)$$

## 11.2 Dipól-dipól interakcia - sila

$$\vec{B}_1\Big|_2 = \frac{\mu_0}{4\pi}\frac{1}{r_{12}^3}\left(\frac{3\vec{r}_{12}(\vec{r}_{12}\cdot\vec{m}_1)}{r_{12}^2} - \vec{m}_1\right)$$

$$\vec{B}_1\cdot m_2 = \frac{\mu_0}{4\pi}\frac{1}{r_{12}^3}\left(3\frac{(\vec{r}_{12}\cdot\vec{m}_2)(\vec{r}_{12}\cdot\vec{m}_1)}{r_{12}^2} - \vec{m}_1\cdot\vec{m}_2\right) = \frac{\mu_0}{4\pi}\frac{1}{r_{12}^3}\mathcal{M}$$

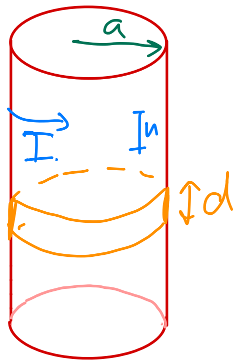
$$\vec{F} = \nabla(\vec{B}\cdot\vec{m}_2) = \frac{\mu_0}{4\pi}\left(\mathcal{M}\,\underbrace{\nabla\frac{1}{r_{12}^3}}_{-3\frac{\vec{r}}{r_{12}^5}} + \frac{1}{r_{12}^3}\nabla\mathcal{M}\right)$$

$$\nabla\mathcal{M} = \partial_i\left(3\frac{x_j m_j^2 x_\ell m_\ell^1}{r^2} - \cancel{m_k^1 m_k^2}\xrightarrow{} 0\right) = 3(\vec{r}_{12}\cdot\vec{m}_2)(\vec{r}_{12}\cdot\vec{m}_1)\overbrace{\nabla\frac{1}{r^2}}^{-2\frac{\vec{r}}{r^4}}$$
$$+\frac{3}{r^2} m_j^2 m_\ell^1\left(\delta_{ij}x_\ell + \delta_{i\ell}x_j\right) =$$

$$= -6(\vec{r}_{12}\cdot\vec{m}_2)(\vec{r}_{12}\cdot\vec{m}_1)\frac{\vec{r}}{r^4} + \frac{3}{r^2}\left[\vec{m}_2(\vec{m}_1\cdot\vec{r}_{12}) + \vec{m}_1(\vec{m}_2\cdot\vec{r}_{12})\right]$$

$$\vec{F} = \frac{\mu_0}{4\pi}\left(-15(\vec{r}_{12}\cdot\vec{m}_2)(\vec{r}_{12}\cdot\vec{m}_1)\frac{\vec{r}}{r^7} + \frac{3}{r^5}\left[\vec{m}_2(\vec{m}_1\cdot\vec{r}_{12}) + \vec{m}_1(\vec{m}_2\cdot\vec{r}_{12})\right]\right.$$
$$\left. + 3\frac{\vec{r}}{r^5}\vec{m}_1\cdot\vec{m}_2\right)$$

# 12.1 Samoindukčnost solenoidu




$$LB = \mu_0 N I \Rightarrow B = \mu_0 n I$$

$$W_M = \frac{1}{2\mu_0}\int B^2\,dV = \frac{\mu_0}{2} n^2 I^2 \int dV = \frac{\mu_0}{2} n^2 I^2 \pi a^2 d$$

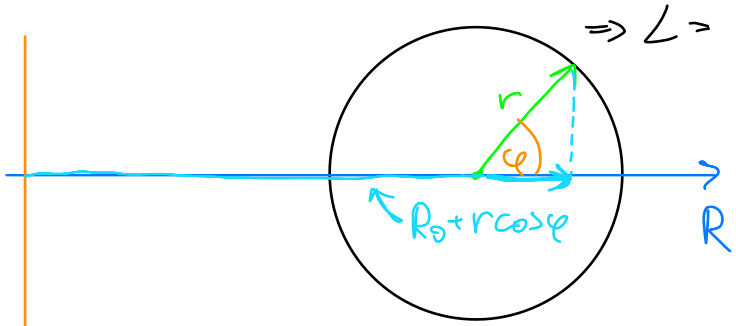
$$\Rightarrow L = \mu_0 n^2 \pi a^2 d \Rightarrow \ell = \mu_0 n^2 a^2 \pi$$

# 12.2 Samoindukčnost toroidálního solenoidu

$$\vec{B} = \frac{I\mu_0}{2\pi R}\vec{e}_\varphi$$

$$W = \frac{1}{2\mu_0}\int B^2\,dV = \frac{\mu_0 I^2}{8\pi^2}\iint \frac{1}{R^2}\,d\ell\,dS = \frac{\mu_0 I^2}{8\pi^2}\int \frac{2\pi R}{R^2}\,dS$$

$$\Rightarrow L = \frac{\mu_0}{2\pi}\int \frac{1}{R}\,dS$$




a) kruh

$$L_{kruh} = \frac{\mu_0}{2\pi}\int \frac{1}{R}\,dS = \frac{\mu_0}{2\pi}\int_0^{a_0}\int_0^{2\pi} \frac{r}{R_0 + r\cos\varphi}\,dr\,d\varphi = \left| \begin{array}{l} u = R_0 + r\cos\varphi \\ du = dr\cos\varphi \end{array}\right.$$

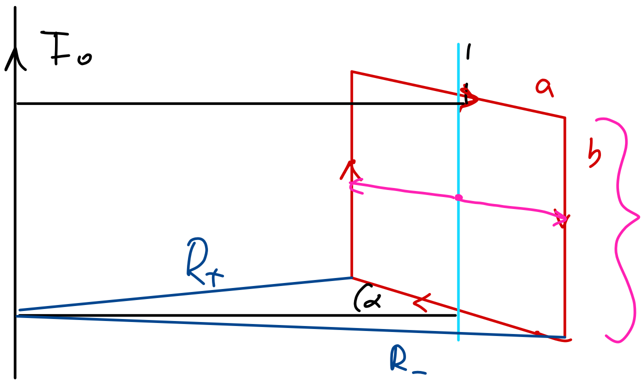
$$= \frac{\mu_0}{2\pi}\int_0^{2\pi}\int_0^{a_0} \frac{\frac{u-R_0}{\cos\varphi}\,\frac{du}{\cos\varphi}}{u}\,d\varphi =$$

$$= \frac{\mu_0}{2\pi}\int_0^{2\pi} \frac{1}{\cos^2\varphi}\int_{R_0}^{R_0 + a_0\cos\varphi} 1 - \frac{R_0}{u}\,du\,d\varphi = \frac{\mu_0}{2\pi}\int_0^{2\pi} \frac{1}{\cos^2\varphi}\left(a_0\cos\varphi - R_0 \ln\left(1 + \frac{a_0\cos\varphi}{R_0}\right)\right)$$

c) obdelník

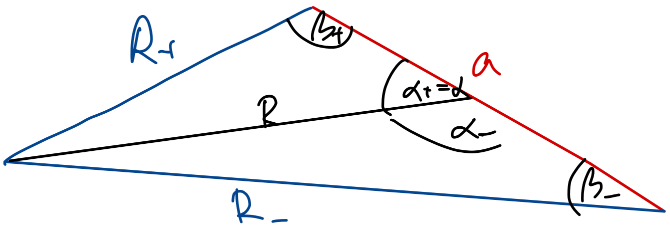
$$L_{obdelník} = \frac{\mu_0}{2\pi}\int_0^d\int_a^b \frac{1}{R}\,dR\,dz = \frac{\mu_0 d}{2\pi}\ln\frac{b}{a}$$

# Moment sily pôsobiaci na obdelníkovú smyčku




$$d\vec{s} = (0, 0, dz)$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I_0}{R}\vec{e}_\varphi$$




$$\frac{\sin\beta_\pm}{R} = \frac{\sin\alpha_\pm}{R_\pm} \to \sin\beta_\pm = \frac{\sin\alpha_\pm R}{R_\pm}$$

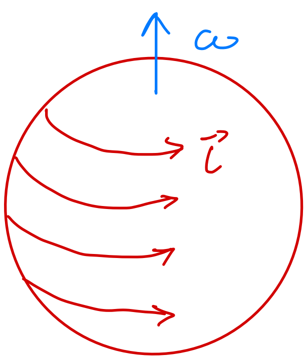
$$\vec{F} = I\int d\vec{s}\times\vec{B} = \frac{\mu_0 I_0 I}{2\pi R_\pm}\int_0^b \vec{e}_\varphi \times \vec{e}_z = \mp\frac{\mu_0 I_0 I b}{2\pi R_\pm}\vec{e}_R$$

$$\vec{M}_\pm = \vec{d}_\pm \times \vec{F}_\pm = \frac{a}{2}F_\pm \sin\beta_\pm \vec{e}_z = \mp\frac{a}{2}\frac{\mu_0 I_0 I b}{2\pi}\frac{\sin\alpha\, R}{R_\pm^2}$$

$$R_\pm^2 = R^2 + \frac{a^2}{4} - Ra\cos\alpha_\pm = R^2 + \frac{a^2}{4} \mp Ra\cos\alpha$$

$$\vec{M} = \vec{M}_+ + \vec{M}_- = \frac{a}{2}\frac{\mu_0}{2\pi} ab\, I_0 I \sin\alpha\left(\frac{R}{R_+^2} + \frac{R}{R_-^2}\right)\vec{e}_z$$

# Rotujúca nabitá guľa




$$\iota = \sigma v = \sigma\omega R = \frac{Q}{4\pi R^2}R\omega = \frac{Q\omega}{4\pi R}$$

$$\vec{m} = \frac{1}{2}\int \vec{x}\times\vec{j}\, dV = \frac{1}{2}\int \vec{x}\times\vec{\iota}\, dS$$

$$\vec{x} = \vec{e}_r + r\sin\vartheta\,\vec{e}_\varphi + r\,\vec{e}_\vartheta$$

$$\vec{\iota} = \frac{Q\omega}{4\pi R}\vec{e}_\varphi$$