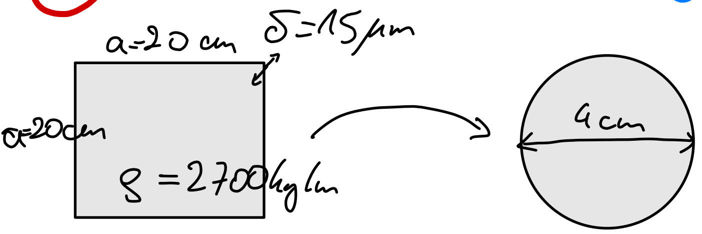
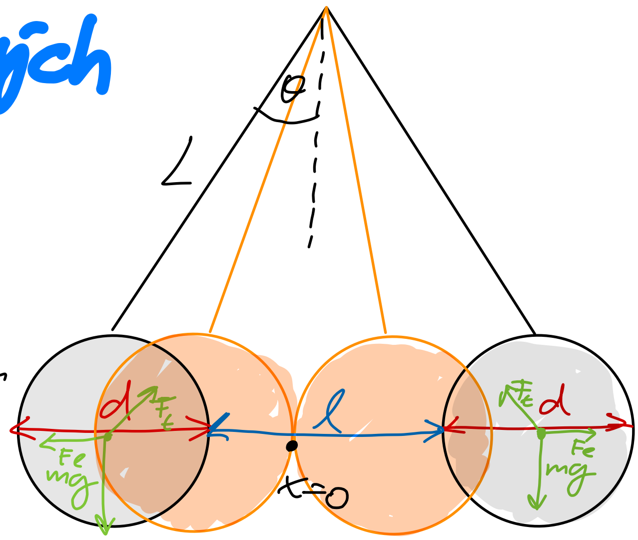
1

Samuel Jankových





$$m = \rho V = \rho a^2 \cdot \delta$$

$$m \approx 1{,}62\ g$$

$$r = \frac{d}{2} = 2cm = 0{,}02m$$
$$d = 4cm = 0{,}04m$$
$$l = 6cm = 0{,}06m$$
$$L = 1m$$

a) Coulomb:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(l+2r)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(l+d)^2}$$

$$F = F_g \sin\theta = mg\sin\theta$$

$$\frac{r+\frac{l}{2}}{L} = \frac{2r+l}{2L} = \frac{d+l}{2L}$$

$$F_e = F \Rightarrow mg\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(l+d)^2}$$

$$\Rightarrow q = \pm\sqrt{mg\frac{(d+l)^3}{2L}4\pi\varepsilon_0} \approx \pm 29{,}7nC$$

2 riešenia: buď sú oba kladné alebo záporné

$$\vec{Q} = \begin{pmatrix} q \\ q \end{pmatrix} = \sum C_{AB} U_B = \overleftrightarrow{C}\vec{U}$$

$$C_{AB} = 4\pi\varepsilon_0 \begin{pmatrix} r & -\frac{r^2}{l+d} \\ -\frac{r^2}{l+d} & r \end{pmatrix} = 4\pi\varepsilon_0 r \begin{pmatrix} 1 & -\frac{r}{l+d} \\ -\frac{r}{l+d} & 1 \end{pmatrix}$$

$$\Rightarrow \vec{U} = \begin{pmatrix} \varphi \\ \varphi \end{pmatrix} = C_{AB}^{-1}\vec{Q} = \frac{1}{4\pi\varepsilon_0 r}\frac{1}{1-\left(\frac{r}{l+d}\right)^2}\begin{pmatrix} 1 & \frac{r}{l+d} \\ \frac{r}{l+d} & 1 \end{pmatrix}\begin{pmatrix} q \\ q \end{pmatrix}$$

$$\Rightarrow \varphi = \frac{1}{4\pi\varepsilon_0 r}\frac{q}{1-\left(\frac{r}{l+d}\right)^2}\left(1+\frac{r}{l+d}\right) = \frac{q}{4\pi\varepsilon_0 r}\frac{1}{1-\frac{r}{l+d}} = \varphi = \frac{q}{4\pi\varepsilon_0 r}\frac{l+d}{l+r} = \pm 16{,}7\,kV$$

b) minimum potenciálnej energie

$$W_{el} = \frac{1}{2}\sum C_{AB} U_A U_B = \frac{1}{2}\vec{U}\overleftrightarrow{C}\vec{U}$$

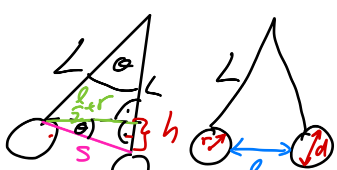
$\rightarrow$ zo symetrie $\vec{U} = \begin{pmatrix} \varphi \\ \varphi \end{pmatrix}$

$$\vec{Q} = \begin{pmatrix} q \\ q \end{pmatrix} = \sum C_{AB} U_B = \overleftrightarrow{C}\vec{U}$$

$$C_{AB} = 4\pi\varepsilon_0 \begin{pmatrix} r & -\frac{r^2}{l+d} \\ -\frac{r^2}{l+d} & r \end{pmatrix} = 4\pi\varepsilon_0 r \begin{pmatrix} 1 & -\frac{r}{l+d} \\ -\frac{r}{l+d} & r \end{pmatrix}$$

$$W_g = 2mgh$$

$$(L-h)^2 + \left(\frac{l}{2}+r\right)^2 = L^2$$
$$\Downarrow$$
$$h = L - \sqrt{L^2 - \left(\frac{l}{2}+r\right)^2}$$



$$h'(l) = \frac{l+d}{4\sqrt{L^2 - \left(\frac{l}{2}+r\right)^2}}$$

$$W_{tot} = W_g - W_{el} = 2mgh(\ell) - 4\pi\varepsilon_0 r\varphi^2 \begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1 & -\frac{r}{\ell+d}\\ -\frac{r}{\ell+d} & r\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} =$$

$$= 2mgh(\ell) - 4\pi\varepsilon_0 r\varphi^2 \begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1-\frac{r}{\ell+d} & 1-\frac{r}{\ell+d}\end{pmatrix} = 2mgh(\ell) - 8\pi\varepsilon_0 r\varphi^2\left(1-\frac{r}{\ell+d}\right)$$

$$\frac{\partial W_{tot}}{\partial \ell} = 2mg\frac{\ell+d}{4\sqrt{L^2-\left(\frac{\ell}{2}+r\right)^2}} - 8\pi\varepsilon_0 r\varphi^2 \frac{r}{(\ell+d)^2} = 0$$

$$\Rightarrow \quad \varphi = \pm\sqrt{\frac{1}{16\pi\varepsilon_0}\frac{mg}{\sqrt{L^2-\left(\frac{\ell}{2}+r\right)^2}}\frac{(\ell+d)^3}{r^2}} \approx \pm 9{,}45\ \text{kV}$$

↑ opäť máme dve riešenia

$$\begin{pmatrix}q\\q\end{pmatrix} = 4\pi\varepsilon_0 r\begin{pmatrix}1 & -\frac{r}{\ell+d}\\ -\frac{r}{\ell+d} & r\end{pmatrix}\begin{pmatrix}\varphi\\\varphi\end{pmatrix} \Rightarrow q = 4\pi\varepsilon_0 r\left(1-\frac{r}{\ell+d}\right)\varphi$$

$$\Rightarrow q = \pm 4\pi\varepsilon_0 \cancel{r}\left(1-\frac{r}{\ell+d}\right)\sqrt{\frac{1}{16\pi\varepsilon_0}\frac{mg}{\sqrt{L^2-\left(\frac{\ell}{2}+r\right)^2}}\frac{(\ell+d)^3}{\cancel{r^2}}} =$$

$$q = \pm\left(1-\frac{r}{\ell+d}\right)\sqrt{\pi\varepsilon_0\, mg\frac{(\ell+d)^3}{\sqrt{L^2-\left(\frac{\ell}{2}+r\right)^2}}} \approx \pm 16{,}83\ \text{nC}$$

→ v prípade bodového náboja je maximum v mieste kde sa nachádza (tam je ∞), teda ak máme vodič, tak maxim bude na jeho povrchu, v prípade dvoch vodičov sa to nezmení
↳ ostáva zistiť, kde na povrchu bude maximum

$$|E| = q'\left(\frac{1}{y^2+z^2+\left(x-\frac{\ell}{2}-r\right)^2} + \frac{1}{y^2+z^2+\left(x+\frac{\ell}{2}+r\right)^2}\right) \quad \leftarrow q' = \frac{q}{4\pi\varepsilon_0}$$

→ súradnice y a z nemajú vplyv, lebo niesú obmedzené, BÚNO $y=z=0$

$$E(x) = q'\left(\frac{1}{\left(x-\frac{\ell}{2}-r\right)^2} + \frac{1}{\left(x+\frac{\ell}{2}+r\right)^2}\right)$$

→ z vety o extréme vzhľadom k množine stačí skúmať hraničné hodnoty intervalov ($\frac{dE}{dx}=0$ len pre $x=0$, tam je lok. minimum.)

$$\lim_{x\to\pm\infty} E(x) = 0$$

$$E\left(\pm\frac{\ell}{2}\right) = q'\left(\frac{1}{r^2}+\frac{1}{(\ell+r)^2}\right) \qquad E\left(\pm\left(\frac{\ell}{2}+d\right)\right) = q'\left(\frac{1}{r^2}+\frac{1}{(\ell+3r)^2}\right)$$

$$E\left(\pm\frac{\ell}{2}\right) > E\left(\pm\left(\frac{\ell}{2}+d\right)\right) \Rightarrow E_{max} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r^2}+\frac{1}{(\ell+2r)^2}\right) \approx 0{,}79\ \frac{\text{MV}}{\text{m}}$$

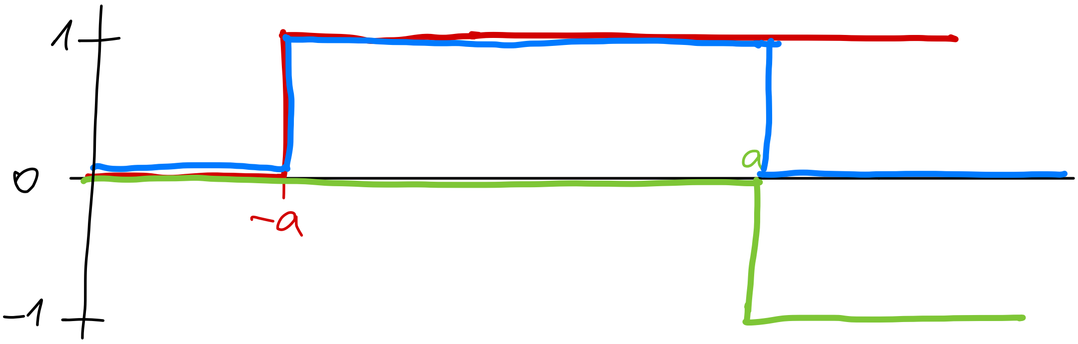
2

$$\phi(x,y,z) = U\int_{-a}^{a} \frac{\sin^2\frac{\pi w}{a}}{\sqrt{x^2+y^2+(z-w)^2}}\,dw = \qquad z'=w$$

$$= U\int_{-a}^{a}\frac{\sin^2\frac{\pi z'}{a}}{\sqrt{x^2+y^2+(z-z')^2}}\,dz' = U\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-a}^{a}\frac{\sin^2\frac{\pi z'}{a}\,\delta(x')\delta(y')}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\,dx'dy'dz' =$$

Heaviside step func.

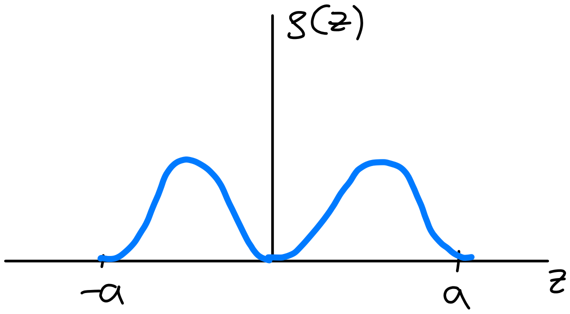
$$= \int_V \frac{U\sin^2\frac{\pi z'}{a}\,\delta(x')\delta(y')\left(\Theta(z'+a)-\Theta(z'-a)\right)}{|\vec r-\vec r'|}\,d\vec r' =$$

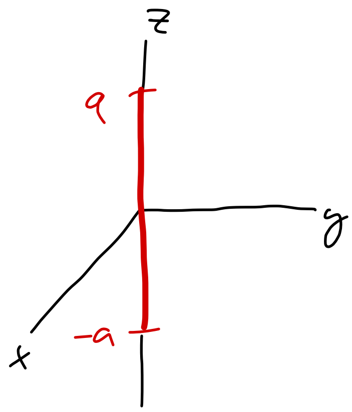


$$= \int_V \frac{f(\vec r')}{|\vec r-\vec r'|}\,d\vec r' \qquad \Rightarrow f(\vec r) = \frac{\rho(\vec r)}{4\pi\varepsilon_0}$$

$$\Rightarrow \rho(\vec r) = 4\pi\varepsilon_0 U\sin^2\left(\frac{\pi z}{a}\right)\delta(x)\,\delta(y)\left(\Theta(z+a)-\Theta(z-a)\right)$$

↳ náboj je v tvare úsečky od $-a$ do a, ale hustota nie je rozložená rovnomerne



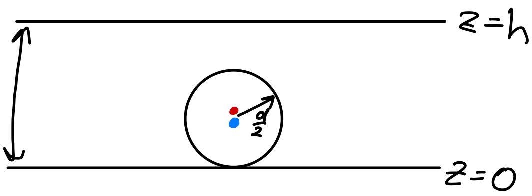


→ celkový náboj je len integrál

$$Q = \int_{-a}^{a} 4\pi\varepsilon_0 U\sin^2\left(\frac{\pi z}{a}\right)dz = 4\pi\varepsilon_0 U a$$

3

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{|\vec{r}-\vec{r}\,'|^3}\vec{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{|\vec{r}-\vec{r}\,''|^3}\vec{r} + \frac{U}{h}$$



→ náboje budú na osi $x=y=0$ $\Rightarrow \vec{r}\,'=(0,0,z_1)$
$\vec{r}\,''=(0,0,z_2)$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(x^2+y^2+(z-z_1)^2)^{\frac{3}{2}}}\vec{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{(x^2+y^2+(z-z_2)^2)^{\frac{3}{2}}}\vec{r} + \frac{U}{h}$$

$$\Rightarrow \varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(x^2+y^2+(z-z_1)^2)^{\frac{1}{2}}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{(x^2+y^2+(z-z_2)^2)^{\frac{1}{2}}} + \frac{Uz}{h}$$

→ zo symetrie môžeme $y=0$

→ plošnú nábojovú hustotu dostaneme ako $\sigma(\vec{r}_s) = \varepsilon_0 \vec{n}\cdot\vec{E}(\vec{r}_s) = \varepsilon_0 E_n$

→ zavedieme súradnice $x = R\sin\theta$ $\quad z = R(1+\cos\theta)$:

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{\left(R^2\sin^2\theta + R^2 + R^2\cos^2\theta - 2Rz_1(1+\cos\theta) + z_1^2\right)^{\frac{1}{2}}} + \frac{Q_2}{\cdots} + \frac{UR(1+\cos\theta)}{h}$$

$$= \frac{Q_1'}{\left(R^2 + (R-z_1)^2 - 2Rz_1\cos\theta\right)^{\frac{1}{2}}} + \frac{Q_2'}{\left(R^2 + (R-z_2)^2 - 2Rz_2\cos\theta\right)^{\frac{1}{2}}} + \frac{UR(1+\cos\theta)}{h}$$

→ analogicky pre $\vec{E}$:

$$\vec{E} = \frac{Q_1'\,\vec{e}_R}{R^2+(R-z_1)^2-2Rz_1\cos\theta} + \frac{Q_2'\,\vec{e}_R}{R^2+(R-z_2)^2-2Rz_2\cos\theta} + \frac{U}{h}$$

$$\Rightarrow E_n(R=\tfrac{d}{2}) = \vec{E}\cdot\vec{e}_R = \frac{Q_1'}{\frac{d^2}{4}+(\frac{d}{2}-z_1)^2-dz_1\cos\theta} + \frac{Q_2'}{\frac{d^2}{4}+(\frac{d}{2}-z_2)^2-dz_2\cos\theta} + \frac{U}{h}$$

$$\Rightarrow \sigma(\theta) = \varepsilon_0 E_n = \varepsilon_0 \vec{E}\cdot\vec{e}_r$$

$$= \frac{1}{4\pi}\left(\frac{Q_1}{\frac{d^2}{4}+(\frac{d}{2}-z_1)^2-dz_1\cos\theta} + \frac{Q_2}{\frac{d^2}{4}+(\frac{d}{2}-z_2)^2-dz_2\cos\theta} + \frac{U}{h}\right)$$

→ z podm. ekvipotenciály musí platiť $\varphi(R=\frac{d}{2}, \theta=0) = \varphi(R=\frac{d}{2}, \theta=\frac{2\pi}{3}) = 0$

↳ to nám dá dve riešenia pre Q_1' a Q_2':

```
Phi[R_, θ_] := Q1 / Sqrt[R^2 + (R - z1)^2 - 2*R*z1*Cos[θ]] +
  Q2 / Sqrt[R^2 + (R - z2)^2 - 2*R*z2*Cos[θ]]  +  U*R*(1 + Cos[θ]) / h
Simplify[Solve[{Phi[d/2, 0] == 0, Phi[d/2, 2*π/3] == 0},  {Q1, Q2}]]
```

$$\left\{\left\{Q1 \to -\frac{d\,U\sqrt{\frac{d^2}{2}-2\,d\,z1+z1^2}\;\sqrt{d^2-d\,z1+2\,z1^2}\;\left(4\sqrt{d^2-4\,d\,z2+2\,z2^2}-\sqrt{d^2-d\,z2+2\,z2^2}\right)}{4\,h\left(\sqrt{d^2-d\,z1+2\,z1^2}\;\sqrt{d^2-4\,d\,z2+2\,z2^2}-\sqrt{d^2-4\,d\,z1+2\,z1^2}\;\sqrt{d^2-d\,z2+2\,z2^2}\right)},\right.\right.$$

$$\left.\left.Q2 \to \frac{d\,U\left(4\sqrt{d^2-4\,d\,z1+2\,z1^2}-\sqrt{d^2-d\,z1+2\,z1^2}\right)\sqrt{\frac{d^2}{2}-2\,d\,z2+z2^2}\;\sqrt{d^2-d\,z2+2\,z2^2}}{4\,h\left(\sqrt{d^2-d\,z1+2\,z1^2}\;\sqrt{d^2-4\,d\,z2+2\,z2^2}-\sqrt{d^2-4\,d\,z1+2\,z1^2}\;\sqrt{d^2-d\,z2+2\,z2^2}\right)}\right\}\right\}$$

→ pre tri náboje budeme mať pole

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(x^2+y^2+(z-z_1)^2)^{\frac{1}{2}}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{(x^2+y^2+(z-z_2)^2)^{\frac{1}{2}}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{(x^2+y^2+(z-z_3)^2)^{\frac{1}{2}}} + \frac{Uz}{h}$$

→ opäť položíme $y=0$ a zavedieme $x = R\sin\theta$ $z = R(1+\cos\theta)$:

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\sqrt{(R-z_1+R\cos\theta)^2+R^2\sin^2\theta}} \quad \frac{Q_2}{\sqrt{(R-z_2+R\cos\theta)^2+R^2\sin^2\theta}}\right.$$

$$\left.\frac{Q_3}{\sqrt{(R-z_3+R\cos\theta)^2+R^2\sin^2\theta}}\right) + \frac{UR(1+\cos\theta)}{h}$$

→ daný potenciál dáme do Mathematiky a vyriešime lin. rovnice:

$$\phi\left(\frac{d}{2};0\right)=0 \qquad \phi\left(\frac{d}{2},\frac{2\pi}{3}\right)=0 \qquad \phi\left(\frac{d}{2},\frac{\pi}{3}\right)=0$$

z toho dostaneme hodnoty

$$Q_1 = 209{,}7\ nC \qquad Q_2 = -217{,}4\ nC \qquad Q_3 = 2{,}6\ nC$$

→ správnosť hodnôt môžeme overiť aj numerickou minimalizáciou integrálu

$$\int_0^{\pi} \left|\varphi\left(\frac{d}{2};\theta\right)\right|^2 d\theta$$, kde minimalizujeme kvadrát

→ z tejto minimalizácie dostaneme

$$Q_1 = 212{,}7\,nC \quad Q_2 = -218{,}6\,nC \quad Q_3 = 0{,}92\ nC$$

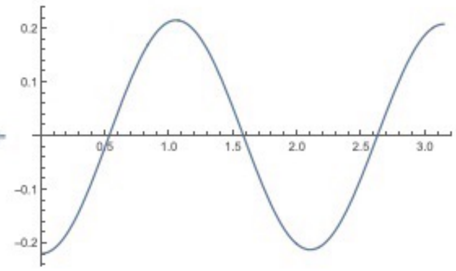
→ hodnoty sú približne rovnaké, ale keďže potenciál nesplňa hraničné podm. tak manuálnym nastavovaním nájdeme správne hodnoty

$$Q_1 = 209\,nC \qquad Q_2 = -218{,}6\ nC \qquad Q_3 = 0{,}8\ nC$$

→ zvolili sme $z_1 = 1{,}73\,cm$ $z_2 = 1{,}77\,cm$ $z_3 = 1{,}7\,cm$

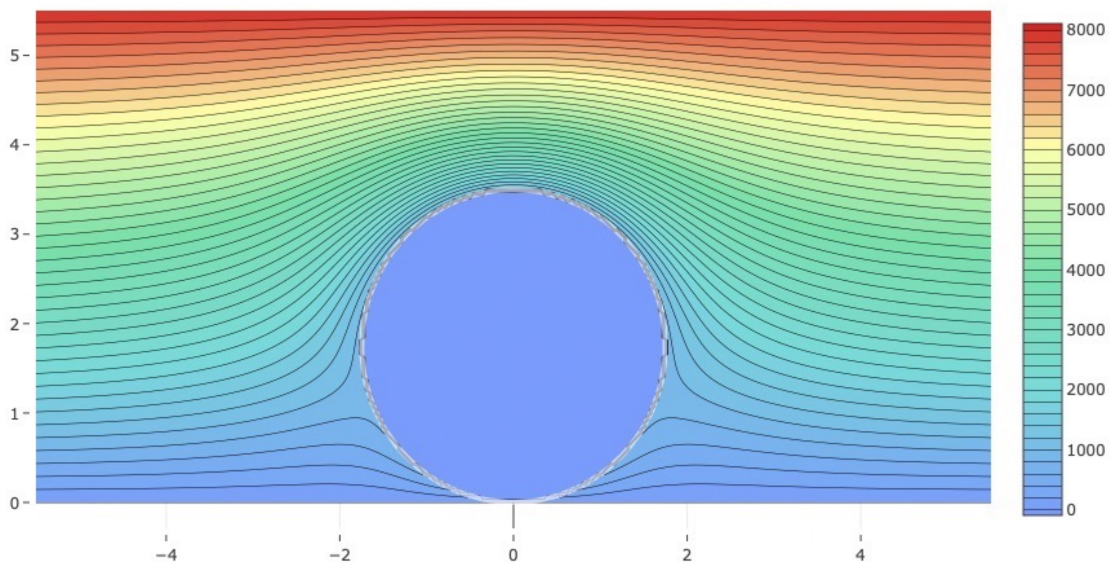
→ skript z Mathematiky:

```
In[392]:= ClearAll["Global`*"]
          Phi[R_, θ_, Q1_, Q2_, Q3_] := Q1 / Sqrt[(R - z1 + R Cos[θ])^2 + R^2 Sin[θ]^2] + Q2 / Sqrt[(R - z2 + R Cos[θ])^2 + R^2 Sin[θ]^2] +
            Q3 / Sqrt[(R - z3 + R Cos[θ])^2 + R^2 Sin[θ]^2] + U * R * (1 + Cos[θ]) / h

          s := Solve[{Phi[d / 2, 0, Q1, Q2, Q3] == 0, Phi[d / 2, 2 * π / 3, Q1, Q2, Q3] == 0, Phi[d / 2, π / 3, Q1, Q2, Q3] == 0},
             {Q1, Q2, Q3}, Reals];
          q1 := s[[All, 1, 2]][[1]];
          q2 := s[[All, 2, 2]][[1]];
          q3 := s[[All, 3, 2]][[1]];

In[464]:= U := 8000
          d := 3.5
          h := 5.5
          z1 := 1.73
          z2 := 1.77
          z3 := 1.70
          e := 8.854 * 10^-12
          q1 * 4 * π * e * 10^9 / 100
          q2 * 4 * π * e * 10^9 / 100
          q3 * 4 * π * e * 10^9 / 100
          min = NMinimize[NIntegrate[(Phi[d / 2, θ, P1 / (4 * π * e * 10^9 / 100), P2 / (4 * π * e * 10^9 / 100), P3 / (4 * π * e * 10^9 / 100)]) ^2,
             {θ, 0, π}], {P1, P2, P3}]

Out[471]= 209.763
Out[472]= -217.36
Out[473]= 2.64079
Out[474]= {0.0717148, {P1 → 212.728, P2 → -218.609, P3 → 0.924725}}

In[491]:= p1 = P1 /. Part[min, 2]
          p2 = P2 /. Part[min, 2]
          p3 = P3 /. Part[min, 2]
          Plot[Phi[d / 2, θ, p1 / (4 * π * e * 10^9 / 100), p2 / (4 * π * e * 10^9 / 100), p3 / (4 * π * e * 10^9 / 100)], {θ, 0, π}]
Out[491]= 212.728
Out[492]= -218.609
Out[493]= 0.924725
Out[494]=
```

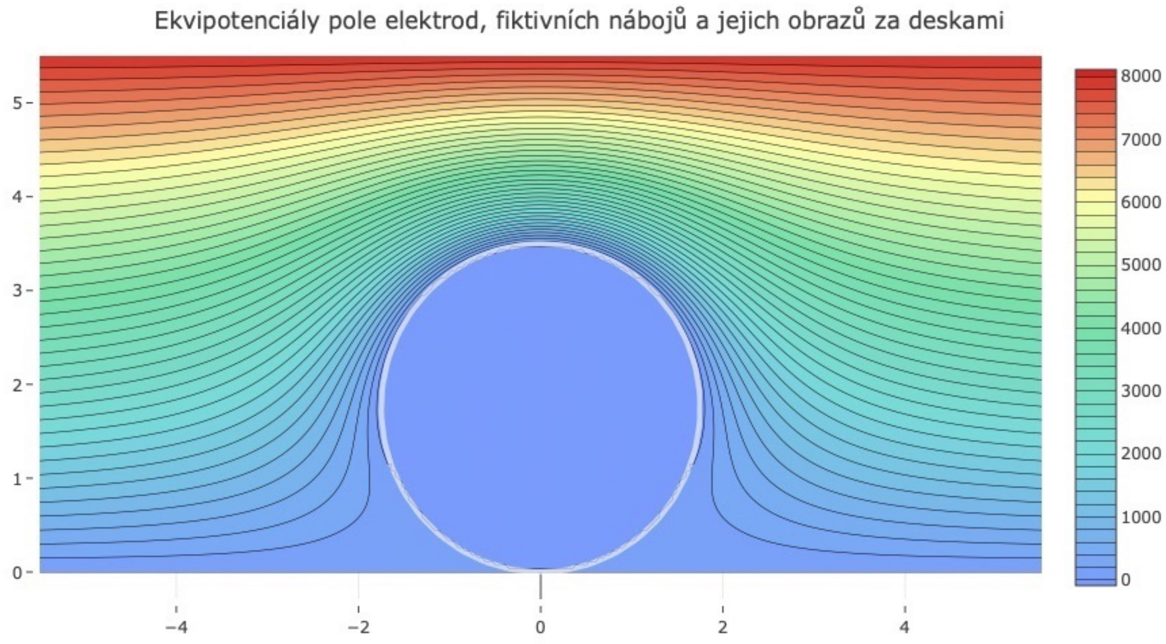


→ graf před manuální minimalizací ($Q_1 = 212{,}7\,nC$, $Q_2 = -218{,}6\,nC$, $Q_3 = 0{,}92\,nC$)

Ekvipotenciály pole elektrod, fiktivních nábojů a jejich obrazů za deskami



→ graf po manuálnej minimalizácii ($Q_1 = 209\,nC$, $Q_2 = -218{,}6\,nC$, $Q_3 = 0{,}8\,nC$)



→ plošnú nábojovú hustotu dostaneme zo vzťahu

$$\sigma(\theta) = \varepsilon_0\, \vec{n}\cdot\vec{E}\left(\frac{d}{2}\right) = \varepsilon_0\, \vec{e}_r\cdot\vec{E}\left(\frac{d}{2}\right) =$$

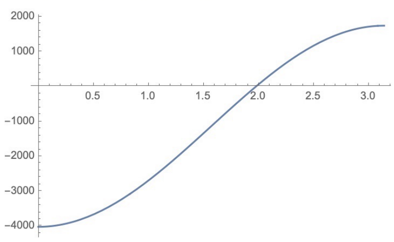
$$= \frac{1}{4\pi}\left(\frac{Q_1}{\left(\frac{d}{2}-z_1+\frac{d}{2}\cos\theta\right)^2+\frac{d^2}{4}\sin^2\theta} + \frac{Q_2}{\left(\frac{d}{2}-z_2+\frac{d}{2}\cos\theta\right)^2+\frac{d^2}{4}\sin^2\theta} \right.$$

$$\left. \frac{Q_3}{\left(\frac{d}{2}-z_3+\frac{d}{2}\cos\theta\right)^2+\frac{d^2}{4}\sin^2\theta} + \frac{U}{h} \right)$$

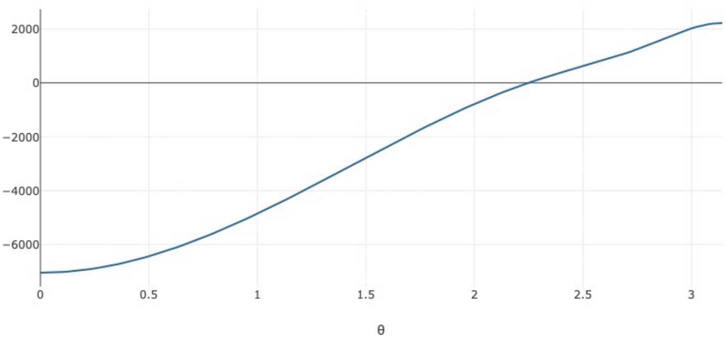
→ graf v dvoch tvaroch (vynesieme iba E_n, tj. $\varepsilon_0 = 1$)

Aproximované

```
Q1 := 209 / (4 * π * ε * 10 ^ 9 / 100)
Q2 := -218.6 / (4 * π * ε * 10 ^ 9 / 100)
Q3 := 0.8 / (4 * π * ε * 10 ^ 9 / 100)
R := d / 2
sigma[θ_] := Q1 / ((R - z1 + R Cos[θ])^2 + R^2 Sin[θ]^2) + Q2 / ((R - z2 + R Cos[θ])^2 + R^2 Sin[θ]^2) +
  Q3 / ((R - z3 + R Cos[θ])^2 + R^2 Sin[θ]^2)  +  U / h

Plot[sigma[θ], {θ, 0, π}]
```



Presnejšie



→ silu tak dostame ako

$$\vec{F} = \int \frac{1}{2}\sigma(\theta)\,\vec{E}\,d\theta = \int \frac{1}{2}\varepsilon_0 E_n(\theta)\,\vec{E}\,d\theta$$

→ zo symetrie nutne $F_x = F_y = 0$

$$|\vec{F}| = F_z = \int \frac{1}{2}\sigma(\theta)\,E_z\,d\Omega = \int_0^{\pi} \frac{\varepsilon_0}{2} E_n(\theta)\,E_z(\theta)\,2\pi R^2 \sin\theta\,d\theta\,d\varphi$$

$$= \int_0^{\pi} \pi\varepsilon_0 E_n(\theta)\,E_z(\theta)\,R^2\sin\theta\,d\theta \approx 1{,}28\,mN$$

U= 8000 (Napětí na horní elektrodě [V])
d= 3.5 (průměr kuličky [cm])
h= 5.5 (vzdálenost desek [cm])

Náboj Q_1
Q_1= 209 [nC]
z_1= 1.73 [cm]

Náboj Q_2
Q_2= -218.6 [nC]
z_2= 1.77 [cm]

Náboj Q_3
Q_3= 0.8 [nC]
z_3= 1.7 [cm]

Nakreslit ekvipotenciály Phi

Funkce f: epsilon*Pi*R*R*sin(theta)*En*Ez
Nakreslit f(r=a,θ): Nakreslit f(x,z=0): Nakreslit f(x,z=h):

θ * (180°/π)	0	30°	60°	90°	120°	150°	180°
Φ(θ)	6.37816707	-49.28209358	-130.73610483	-118.22021578	36.0845453	188.50810506	0
f(θ)	0.00000e+0	1.51542e-3	8.20652e-4	-1.90147e-5	-1.47066e-5	-3.01298e-5	-5.14840e-20
$\int^{\theta}$ f(θ') dθ'	0.00000e+0	4.83310e-4	1.16468e-3	1.33056e-3	1.30607e-3	1.30171e-3	1.27818e-3

x	-10 h	-3 h	-h	0	+h	+3h	+10h
f(x,z=h)	-1.79813e-4	-1.75779e-4	-1.71770e-4	0.00000e+0	-1.71770e-4	-1.75779e-4	-1.79813e-4
$\int^{x}$ f(x',z=h) dx'	9.74951e-3	2.86340e-3	9.75911e-4	0.00000e+0	-9.75911e-4	-2.86340e-3	-9.74951e-3
f(x,z=h)	1.80140e-4	1.79196e-4	1.46107e-4	5.14840e-20	1.46107e-4	1.79196e-4	1.80140e-4
$\int^{x}$ f(x',z=0) dx'	-9.18012e-3	-2.25294e-3	-3.43776e-4	0.00000e+0	3.43776e-4	2.25294e-3	9.18012e-3