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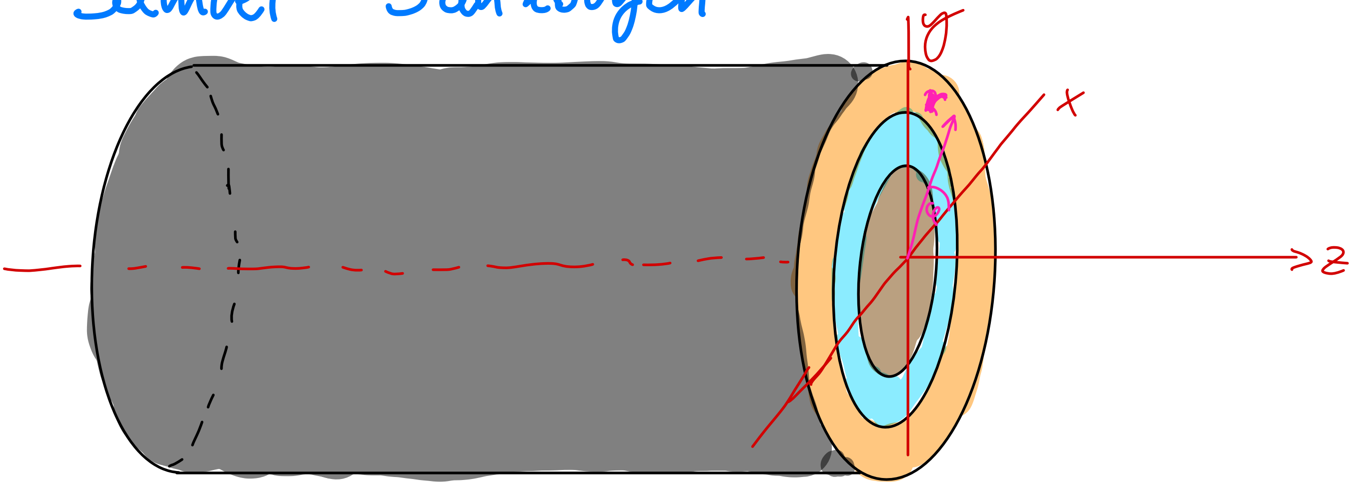

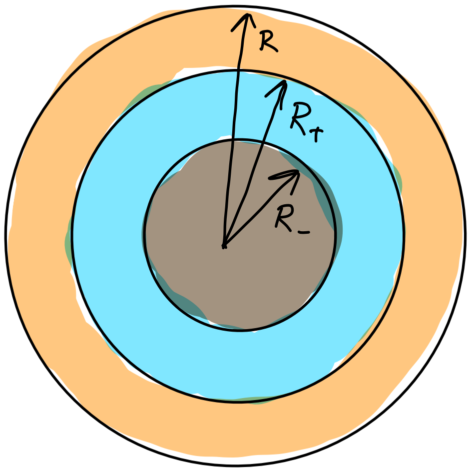

$\phi_-(r,z)$ $\phi_+(r,z)$

$A_-(r,z)$ $A_+(r,z)$

(i)

$A_\pm \vec{e}_z = A_\pm(r,z)\vec{e}_z$ jediné $\partial_z A_z \neq 0 \Rightarrow \vec{B} = \nabla \times \vec{A} = 0$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \Rightarrow E_z = -\partial_z \phi - \partial_t A$$

$$\Rightarrow E_z\big|_{vodič} = 0 \Rightarrow -\partial_z \phi_\pm = \partial_t A_\pm$$

(ii)

$$\left.\begin{aligned} -\frac{1}{c^2}\partial_t^2 \phi_\pm + \partial_z^2 \phi_\pm &= 0 \\ -1/c^2\, \partial_t^2 A_\pm + \partial_z^2 A_\pm &= 0 \end{aligned}\right\}$$ obecné riešení vlnové rovnice $$\left\{\begin{aligned} \phi_\pm &= f_\pm(z-ct) \\ A_\pm &= \tilde{f}_\pm(z-ct) \end{aligned}\right.$$

→ podmienka vymiznutia $\vec{E}$: $\begin{aligned} \partial_z \phi_\pm &= f_\pm' \\ \partial_t A_\pm &= -c\tilde{f}_\pm' \end{aligned} \Rightarrow \tilde{f}_\pm = \frac{1}{c} f + \cancel{konst}^{\,0}$

(iii)

$$\left.\begin{aligned} \boxed{\phi_\pm = f_\pm(z-ct)} &= -\partial_t \Psi_\pm(z,t) \\ \boxed{A_\pm = \frac{1}{c} f_\pm(z-ct)} &= \partial_z \Psi_\pm(z,t) \end{aligned}\right\} \Psi_\pm(z,t) = \frac{1}{c} F_\pm(z-ct)$$

(iv)

$\Delta\phi = \partial_R^2 \phi + \frac{1}{R}\partial_R \phi + \frac{1}{R^2}\partial_\varphi^2 \phi = 0$ → zo symetrie $\partial_\varphi \phi = \partial_\varphi A = 0$

$$\Rightarrow \phi'' + \frac{1}{R}\phi' = 0 \Rightarrow R\phi'' + \phi' = (R\phi')' = 0$$

$$\Rightarrow R\phi' = c \Rightarrow \phi = c \ln R + d = c' \ln \frac{R}{R_0}$$

↳ analogicky $A = \tilde{c} \ln R + \tilde{d} = \tilde{c}' \ln \frac{R}{R_0}$

$$\phi(R=R_\pm) = c\ln\frac{R_\pm}{R_0} = f_\pm(z-ct) = \begin{cases} c\ln\frac{R_+}{R_0} = f_+(z-ct) \\ c\ln\frac{R_-}{R_0} = f_-(z-ct) \end{cases}$$

$$c\left(\ln\frac{R_+}{\cancel{R_0}} - \ln\frac{R_-}{\cancel{R_0}}\right) = f_+ - f_-$$

$$\Rightarrow c = \frac{1}{\ln\frac{R_+}{R_-}}(f_+ - f_-)$$

$$\Rightarrow \ln\frac{R_+}{R_0}(f_+ - f_-) = \ln\frac{R_+}{R_-} f_+$$

$$\ln R_+ - \ln R_0 = \ln\frac{R_+}{R_-}\frac{f_+}{f_+ - f_-}$$

$$\Rightarrow \ln R_0 = \ln R_+ - \ln R_+ \frac{f_+}{f_+ - f_-} + \ln R_- \frac{f_+}{f_+ - f_-} =$$

$$= \ln R_- \frac{f_+}{f_+ - f_-} - \ln R_+ \frac{f_-}{f_+ - f_-}$$

$$\Rightarrow \phi(R,z,t) = \frac{f_+ - f_-}{\ln\frac{R_+}{R_-}}\ln R - \frac{f_+\ln R_- - f_-\ln R_+}{\ln\frac{R_+}{R_-}} = \frac{f_+(z-ct)\ln\frac{R}{R_-} - f_-(z-ct)\ln\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}$$

analogicky

$$A(R,z,t) = \frac{1}{c}\,\frac{f_+(z-ct)\ln\frac{R}{R_-} - f_-(z-ct)\ln\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}$$

Kontrola:

$$\partial_R\phi = \frac{f_+(z-ct)\frac{1}{R} - f_-(z-ct)\frac{1}{R}}{\ln\frac{R_+}{R_-}} = \frac{1}{R}\left(\frac{f_+ - f_-}{\ln\frac{R_+}{R_-}}\right)$$

$$\partial_R^2\phi = -\frac{1}{R^2}\left(\frac{f_+ - f_-}{\ln\frac{R_+}{R_-}}\right)$$

$$\partial_z^2\phi = \frac{f_+''\ln\frac{R}{R_-} - f_-''\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}$$

$$\partial_t^2\phi = \frac{c^2 f_+''\ln\frac{R}{R_-} - c^2 f_-''\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}$$

$$\partial_\varphi^2\phi = 0$$

$$\Box\phi = -\frac{1}{c^2}\partial_t^2\phi + \partial_z^2\phi + \partial_R^2\phi + \frac{1}{R}\partial_R\phi + \frac{1}{R^2}\partial_\varphi^2\phi$$

$$= \cancel{\frac{f_+''\ln\frac{R}{R_-} - f_-''\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}} + \cancel{\frac{f_+''\ln\frac{R}{R_-} - f_-''\frac{R}{R_+}}{\ln\frac{R_+}{R_-}}}$$

$$-\frac{1}{R^2}\cancel{\left(\frac{f_+ - f_-}{\ln\frac{R_+}{R_-}}\right)} + \frac{1}{R^2}\cancel{\left(\frac{f_+ - f_-}{\ln\frac{R_+}{R_-}}\right)} + 0 = 0$$

→ analogicky pre A

(v)

$$\vec{B} = \nabla \times \vec{A} = \left(\cancel{\frac{1}{R}\partial_\varphi A_z} - \cancel{\partial_z A_\varphi}\right)\vec{e}_R + \left(\cancel{\partial_z A_R} - \partial_R A_z\right)\vec{e}_\varphi + \frac{1}{R}\left(\cancel{\partial_R (R A_\varphi)} - \cancel{\partial_\varphi A_R}\right)\vec{e}_z$$

$$\Rightarrow \vec{B} = -\partial_R A_z \vec{e}_\varphi = -\partial_R A \vec{e}_\varphi = \frac{-1}{cR}\left(\frac{f_+ - f_-}{\ln \frac{R_+}{R_-}}\right)\vec{e}_\varphi = \vec{B}$$

$$\tilde{\Psi}(z,t) = A_+(z,t) - A_-(z,t) = \frac{1}{c} f_+(z-ct) - \frac{1}{c} f_-(z-ct)$$

$$\Rightarrow \vec{B} = -\frac{1}{R} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}} \vec{e}_\varphi$$

$$\vec{E} = -\partial_R \phi \vec{e}_R - \partial_z \phi \vec{e}_z - \partial_t A \vec{e}_z =$$

$$\vec{E} = -\frac{1}{R}\left(\frac{f_+ - f_-}{\ln \frac{R_+}{R_-}}\right)\vec{e}_R - \cancel{\frac{f_+' \ln \frac{R}{R_-} - f_-' \ln \frac{R}{R_+}}{\ln \frac{R_+}{R_-}}}\vec{e}_z + \cancel{\frac{f_+' \ln \frac{R}{R_-} - f_-' \ln \frac{R}{R_+}}{\ln \frac{R_+}{R_-}}}\vec{e}_z$$

$$V(z,t) = \phi_+(z,t) - \phi_-(z,t) = V(z,t)$$

$$\vec{E} = -\frac{1}{R} \frac{V}{\ln \frac{R_+}{R_-}} \vec{e}_R$$

(vi)

$$\vec{n}_\pm \cdot \vec{E}\Big|_{R=R_\pm} = \pm \frac{1}{R_\pm} \frac{V}{\ln \frac{R_+}{R_-}} = \frac{1}{\varepsilon_0} \sigma_\pm$$

$$\Rightarrow \lambda_\pm = \pm \frac{2\pi \cancel{R_\pm} \varepsilon_0}{\cancel{R_\pm}} \frac{V}{\ln \frac{R_+}{R_-}} = \pm \frac{2\pi V \varepsilon_0}{\ln \frac{R_+}{R_-}} = \pm \lambda$$

$$\lambda = \frac{2\pi V \varepsilon_0}{\ln \frac{R_+}{R_-}} \Rightarrow \tilde{C} = \frac{2\pi \varepsilon_0}{\ln \frac{R_+}{R_-}}$$

$$\vec{n}_\pm \times \vec{E} = \mp (\dots) \overbrace{\vec{e}_R \times \vec{e}_R}^{0} = 0$$

(vii)

$$\vec{B} = -\frac{1}{R} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}} \vec{e}_\varphi \qquad \vec{n}_\pm \cdot \vec{B} = \mp (\dots) \vec{e}_R \cdot \vec{e}_\varphi = 0$$

$$\vec{n}_\pm \times \vec{B} = \pm \vec{e}_R \times \vec{e}_\varphi \left(\frac{1}{R_\pm} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}}\right) = \pm \frac{1}{cR_\pm} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}} \vec{e}_z = \frac{1}{\varepsilon_0 c^2} \vec{\mathcal{I}}_\pm$$

$$\Rightarrow \vec{\mathcal{I}}_\pm = \pm \frac{\varepsilon_0 c^2}{R_\pm} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}} \vec{e}_z \qquad \Rightarrow \mathcal{I}_\pm = \pm \frac{\varepsilon_0 c^2}{R_\pm} \frac{\tilde{\Psi}}{\ln \frac{R_+}{R_-}}$$

$$I_\pm = 2\pi R_\pm L_\pm = \pm \frac{2\pi\varepsilon_0 c^2}{\ln\frac{R_+}{R_-}}\tilde{\psi} = \pm I \quad , \text{kde} \quad I = \frac{2\pi\varepsilon_0 c^2}{\ln\frac{R_+}{R_-}}\tilde{\psi}$$

$$\Rightarrow \tilde{L} = \frac{\ln\frac{R_+}{R_-}}{2\pi\varepsilon_0 c^2}$$

(viii)

$$\left.\begin{aligned}\tilde{\psi} &= \frac{1}{c}(f_+ - f_-)\\ \tilde{V} &= f_+ - f_-\end{aligned}\right\} \tilde{\psi} = \frac{1}{c}\tilde{V} \quad \Rightarrow I = \frac{2\pi\varepsilon_0 c}{\ln\frac{R_+}{R_-}} V \Rightarrow \tilde{R} = \frac{\ln\frac{R_+}{R_-}}{2\pi\varepsilon_0 c}$$

energia je odvedená vlnou vo vlnovode

$$\frac{1}{\sqrt{\tilde{L}\tilde{C}}} = c \qquad \sqrt{\frac{\tilde{L}}{\tilde{C}}} = \frac{\ln\frac{R_+}{R_-}}{2\pi\varepsilon_0 c} = \tilde{R}$$

(ix)

$\tilde{\psi} = \frac{1}{c}\tilde{V}$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2 = \frac{\varepsilon_0}{2R^2}\frac{\tilde{V}^2}{\ln^2\frac{R_+}{R_-}} + \varepsilon_0 c^2 \frac{1}{2R^2}\frac{\tilde{\psi}^2}{\ln^2\frac{R_+}{R_-}} = \frac{\varepsilon_0 \tilde{V}^2}{2R^2\ln^2\frac{R_+}{R_-}} + \frac{\varepsilon_0 \tilde{V}^2}{2R^2\ln^2\frac{R_+}{R_-}}$$

$$= \frac{\varepsilon_0 \tilde{V}^2}{R^2\ln^2\frac{R_+}{R_-}} = \frac{1}{c}\frac{I\tilde{V}}{2\pi R^2\ln\frac{R_+}{R_-}}$$

(x)

$$\vec{S} = \varepsilon_0 c^2\, \vec{E}\times\vec{B} = \frac{\varepsilon_0 c^2 \tilde{V}\tilde{\psi}}{R^2\ln^2\frac{R_+}{R_-}}\overbrace{\vec{e}_R\times\vec{e}_\varphi}^{\vec{e}_z} = \frac{\tilde{V}I}{2\pi R^2\ln\frac{R_+}{R_-}}\vec{e}_z = cu\vec{e}_z$$

$$\Rightarrow \vec{v} = c\vec{e}_z$$