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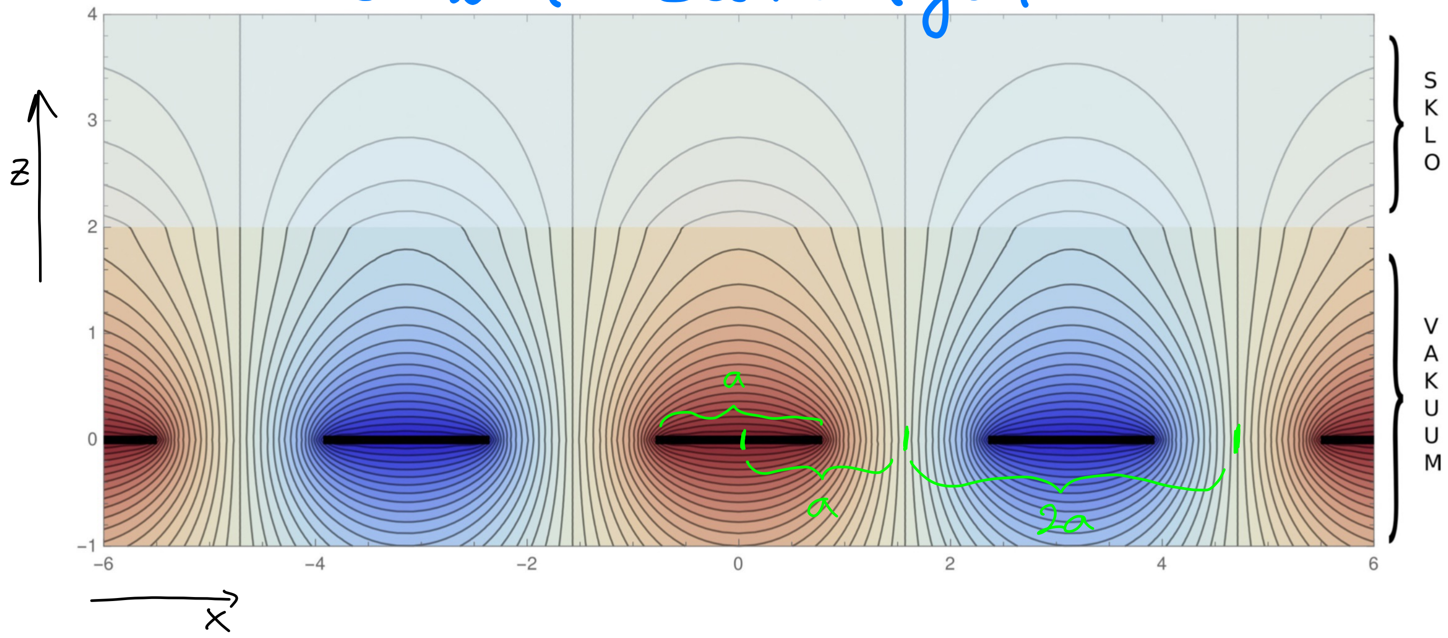


Pole elektród

$\phi_0(x,z) = p\cos(kx)\, h(|z|)$

→ musí spĺňať Laplaca

$$\partial_x^2\phi_0 + \partial_z^2\phi_0 = -\cancel{p}\,k^2\cancel{\cos(kx)}\,h(|z|) + \cancel{p}\,\cancel{\cos(kx)}\,h''(|z|) = 0$$

$$h''(|z|) - k^2 h(|z|) = 0 \Rightarrow h(|z|) = Ae^{k|z|} + Be^{-k|z|}$$

• vyžadujeme nulovosť potenciálu v ∞ : $\lim\limits_{z\to\infty} h(|z|)=0 \Rightarrow A=0$

→ môžeme zvoliť $B=1$ – zahrnuté v p $\Rightarrow$ $h(|z|)=e^{-k|z|}$

$$\Rightarrow \phi_0(x,z) = p\cos(kx)\,e^{-k|z|}$$

• σ_0 dostaneme z Poissonas

$$-\partial_x^2\phi_0 - \partial_z^2\phi_0 = pk^2\cos(kx)\,e^{-k|z|} + kp\cos(kx)\,\partial_z\big(\overbrace{\operatorname{sgn} z}^{2\Theta(z)-1}\, e^{-k|z|}\big) =$$

$$= \cancel{pk^2\cos(kx)e^{-k|z|}} + 2kp\cos kx\,\delta(z)\,e^{-k|z|} - \cancel{pk^2\cos(kx)e^{-k|z|}}$$

$$= 2kp\cos kx\, e^{-k|z|}\delta(z) = 2kp\cos kx\,\delta(z) = \frac{\sigma_0}{\varepsilon_0}$$

$$\Rightarrow \sigma_0(x,z) = 2kp\,\varepsilon_0\cos kx\,\delta(z)$$

• keďže sa elektródy striedajú $+U, -U$, tak nulové hodnoty cos, musia byť v strede medzi elektródami (symetria):

$$\phi_0((2n+1)a,0) = p\cos((2n+1)ak)\,h(0) = 0 \Rightarrow k = \frac{\pi}{2a}$$

$$\phi_0(x,z) = p\cos\left(\frac{\pi x}{2a}\right) e^{-\frac{\pi}{2}\frac{|z|}{a}}$$

• cos je sudá, periodická fce → čo vystihuje symetriu problému

↳ navyše pre $z=0$ a $x \sim (2n+1)a$ je $\cos(kx) \sim 1$, teda v malom okolí stredu elektródy je $\phi_0 \sim p$, teda konšt.

↳ reálne by malo byť $\phi_0 = \text{konšt}$ pre $x \in [-\frac{a}{2}, \frac{a}{2}] + (2n+1)a$

• podľa toho akú podmienku dáme na ϕ_0 dostaneme rôzne hodnoty p:

1) $\phi_0(0,0) = U = p_1$

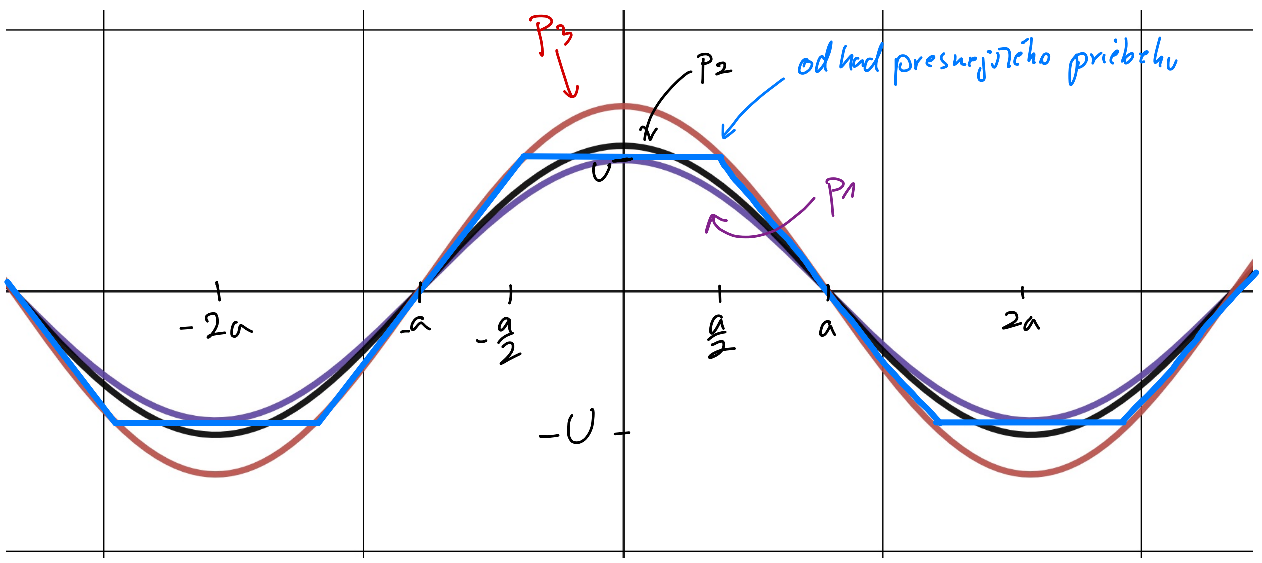
2) $\int_{-\frac{a}{2}}^{\frac{a}{2}} \phi_0(x,0)\,dx = \int_{-\frac{a}{2}}^{\frac{a}{2}} U\,dx = U a$

$$\left[p \frac{\sin\left(\frac{\pi x}{2a}\right)}{\frac{\pi}{2a}}\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{4pa}{\pi}\sin\left(\frac{\pi}{4}\right) = \frac{2\sqrt{2}}{\pi}ap = Ua \quad \Rightarrow p_2 = \frac{\pi}{2\sqrt{2}}U$$

3) $\phi_0(\frac{a}{2};0) = \phi_0(-\frac{a}{2},0) = p\cos\left(\frac{\pi}{4}\right) = U \Rightarrow p_3 = \sqrt{2}U$

⇒ máme teda 3 možné varianty:

$p_1 = U \qquad p_2 = \frac{\pi}{2\sqrt{2}}U \qquad p_3 = \sqrt{2}U$



Rozhraní vakuum-sklo

$$\phi_1(x,z) = q\cos\left(\frac{\pi}{2}\frac{x}{a}\right)e^{-\frac{\pi}{2}\frac{|z-b|}{a}}$$

$$\phi = \phi_0 + \phi_1 = p\cos\left(\frac{\pi}{2}\frac{x}{a}\right)e^{-\frac{\pi}{2}\frac{|z|}{a}} + q\cos\left(\frac{\pi}{2}\frac{x}{a}\right)e^{-\frac{\pi}{2}\frac{|z-b|}{a}}$$

$$\vec{E} = -\nabla\phi = -\nabla\phi_0 - \nabla\phi_1 = \vec{E}_0 + \vec{E}_1$$

$$E_y = 0$$

$$E_x = -\frac{\partial}{\partial x}\phi_0 - \frac{\partial}{\partial x}\phi_1 = \frac{\pi}{2a}\sin\left(\frac{\pi}{2}\frac{x}{a}\right)\left(p\, e^{-\frac{\pi}{2}\frac{|z|}{a}} + q\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}}\right)$$

$$E_z = -\frac{\partial}{\partial z}\phi_0 - \frac{\partial}{\partial z}\phi_1 = \frac{\pi}{2a}\cos\left(\frac{\pi}{2}\frac{x}{a}\right)\left(p\,\mathrm{sgn}(z)\, e^{-\frac{\pi}{2}\frac{|z|}{a}} + q\,\mathrm{sgn}(z-b)\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}}\right)$$

$$[E_x] = 0 \Rightarrow p\lim_{\varepsilon\to 0} e^{-\frac{\pi}{2}\frac{|b+\varepsilon|}{a}} + q\lim_{\varepsilon\to 0} \cancelto{1}{e^{-\frac{\pi}{2}\frac{|\varepsilon|}{a}}} = p\lim_{\varepsilon\to 0} e^{-\frac{\pi}{2}\frac{|b-\varepsilon|}{a}} + q\lim_{\varepsilon\to 0} \cancelto{1}{e^{-\frac{\pi}{2}\frac{|\varepsilon|}{a}}}$$

$$p e^{-\frac{\pi}{2}\frac{b}{a}} + q = p e^{-\frac{\pi}{2}\frac{b}{a}} + q \quad \Rightarrow \quad 0 = 0 \quad \checkmark$$

$[E_y] = 0$ triviálne

$$[D_z] = 0 \Rightarrow p\varepsilon_r \lim_{\varepsilon\to 0} \cancelto{1}{\mathrm{sgn}(b+\varepsilon)}\, e^{-\frac{\pi}{2}\frac{|b+\varepsilon|}{a}} + q\varepsilon_r \lim_{\varepsilon\to 0} \cancelto{1}{\mathrm{sgn}(\varepsilon)}\, \cancelto{1}{e^{-\frac{\pi}{2}\frac{|\varepsilon|}{a}}} =$$

$$= p\lim_{\varepsilon\to 0} \cancelto{1}{\mathrm{sgn}(b-\varepsilon)}\, e^{-\frac{\pi}{2}\frac{|b-\varepsilon|}{a}} + q\lim_{\varepsilon\to 0} \cancelto{-1}{\mathrm{sgn}(-\varepsilon)}\, \cancelto{1}{e^{-\frac{\pi}{2}\frac{|\varepsilon|}{a}}}$$

$$\Rightarrow p\varepsilon_r e^{-\frac{\pi}{2}\frac{b}{a}} + q\varepsilon_r = p e^{-\frac{\pi}{2}\frac{b}{a}} - q \quad \Rightarrow \quad q(\varepsilon_r + 1) = p(1-\varepsilon_r)e^{-\frac{\pi}{2}\frac{b}{a}}$$

$$\Rightarrow q = p\frac{(1-\varepsilon_r)}{(1+\varepsilon_r)}e^{-\frac{\pi}{2}\frac{b}{a}}$$

závisí od presnosti p

$$\Rightarrow \phi_1 = \frac{\pi}{2\sqrt{2}}\frac{1-\varepsilon_r}{1+\varepsilon_r}\cos\left(\frac{\pi}{2}\frac{x}{a}\right)e^{-\frac{\pi}{2}\frac{b}{a}}\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}} = q\cos\left(\frac{\pi}{2}\frac{x}{a}\right)e^{-\frac{\pi}{2}\frac{|z-b|}{a}}$$

$$-\partial_x^2\phi_1(x,b) - \partial_z^2\phi_1(x,b) = q\left(\frac{\pi}{2a}\right)^2\cos\left(\frac{\pi}{2}\frac{x}{a}\right) + \frac{\pi}{2a}q\cos\left(\frac{\pi}{2}\frac{x}{a}\right)\partial_z\left(\overset{2\Theta(z-b)-1}{\mathrm{sgn}(z-b)}\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}}\right)\Bigg|_{z=b}$$

$$= q\left(\frac{\pi}{2a}\right)^2\cos\left(\frac{\pi}{2}\frac{x}{a}\right)\left(\cancel{1} - \cancel{1} + \frac{4a}{\pi}\delta(z-b)\right) =$$

$$= q\frac{\pi}{a}\cos\left(\frac{\pi}{2}\frac{x}{a}\right)\delta(z-b) = \frac{\sigma_1(x,z)}{\varepsilon_0}$$

$$\Rightarrow \sigma_1(x,z) = \varepsilon_0 q\frac{\pi}{a}\cos\left(\frac{\pi}{2}\frac{x}{a}\right)\delta(z-b)$$

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = \left(\frac{\pi}{2a}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{a}\right)\left(p\, e^{-\frac{\pi}{2}\frac{|z|}{a}} + q\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}}\right)$$

pro $z>b$

$$- \left(\frac{\pi}{2a}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{a}\right)\left(p\, \underbrace{\cancel{\operatorname{sgn}(z)}}_{1 \text{ pro } z>b}\, e^{-\frac{\pi}{2}\frac{|z|}{a}} + q\, \underbrace{\cancel{\operatorname{sgn}(z-b)}}_{1}\, e^{-\frac{\pi}{2}\frac{|z-b|}{a}}\right) = 0$$

$$\Rightarrow \varrho = 0$$

$$\sigma = \varepsilon_0 \vec{e}_z \cdot [\vec{E}] = \varepsilon_0 [E_z] = \left(\cancel{p\, e^{-\frac{\pi}{2}\frac{b}{a}}} + q \cancel{- p\, e^{-\frac{\pi}{2}\frac{b}{a}}} + q\right)\varepsilon_0 \frac{\pi}{2a}\cos\left(\frac{\pi}{2}\frac{x}{a}\right) =$$

$$\sigma = \frac{\varepsilon_0 \pi}{a} q \cos\left(\frac{\pi}{2}\frac{x}{a}\right)$$

$$\underbrace{\frac{1}{2}\int_0^{4a} 1 + \cancel{\cos\left(\frac{\pi x}{a}\right)}^{\,0} = \frac{1}{2}}$$

$$E_{0z}(x,b) = \frac{\pi p}{2a}\cos\left(\frac{\pi}{2}\frac{x}{a}\right) e^{-\frac{\pi}{2}\frac{b}{a}}$$

$$\sigma_1 E_{0z} = \varepsilon_0 q p \frac{\pi^2}{2a^2}\cos^2\left(\frac{\pi}{2}\frac{x}{a}\right) \rightarrow \langle \sigma_1 E_{0z}\rangle = \varepsilon_0 p q \frac{\pi^2}{2a^2}\frac{1}{4a}\int_0^{4a}\cos^2\left(\frac{2\pi x}{4a}\right)dx$$

$$\langle \sigma_1 E_{0z}\rangle = \varepsilon_0 p q \frac{\pi^2}{4a^2} = \varepsilon_0 p^2 \frac{\pi^2}{4a}\frac{(1-\varepsilon_r)}{(1+\varepsilon_r)} e^{-\frac{\pi}{2}\frac{b}{a}} \overset{p=p_2}{=} \varepsilon_0 \frac{\pi^4}{32a}\frac{1-\varepsilon_r}{1+\varepsilon_r} e^{-\frac{\pi}{2}\frac{b}{a}}$$

v závislosti od voľby presnosti v p