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- Základný popis
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- Vedenie vlny
- Módy
- Útlm

Nelineárna optika

- 2. rádu
- 3. rádu

Interakce záření s hmotou

- Základný popis
- Fresnelovy koeficienty
- Poyntingov vektor
- Lorentzov model
- Drudeho model

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- Základní popis
- Wienov zákon
- Rayleighův-Jeansův zákon
- Plackov zákon

Elekromagnetické vlny

Maxwellove rovnice

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla\cdot\vec{D} = \rho$$
$$\nabla\cdot\vec{B} = 0 \qquad \nabla\cdot\vec{B} = 0$$
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \nabla\times\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t}$$

Vlnová rovnice

– obecná: $\frac{\partial^2}{\partial x^2}u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u = 0 \quad \longrightarrow \quad \left(\frac{\partial}{\partial x} - \frac{1}{v}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} + \frac{1}{v}\frac{\partial}{\partial t}\right)u = 0$$

označme $\xi_1 = x - vt$ $\qquad \frac{\partial}{\partial t} = -v\frac{\partial}{\partial \xi_1} + v\frac{\partial}{\partial \xi_2}$

$\xi_2 = x + vt$ $\qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi_1} + \frac{\partial}{\partial \xi_2}$

potom $\frac{\partial^2 u}{\partial \xi_1 \partial \xi_2} = 0 \implies u = f(\xi_1) + g(\xi_2) = f(x-vt) + g(x+vt)$

Rovinná vlna

$$\vec{E} = \vec{E}_0\cos(\vec{k}(\vec{r}-\vec{v}t)+\delta) = \vec{E}_0\cos(\vec{k}(\vec{r}-\vec{v}t)+\underbrace{\delta}_{\text{posun fáze}}) = \vec{E}_0\cos\underbrace{(\vec{k}\cdot\vec{r}-\vec{k}\cdot\vec{v}t+\delta)}_{\text{fáze}}$$
$$= \vec{E}_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)$$

$k = \frac{2\pi}{\lambda}$ $\qquad \omega = 2\pi f = \vec{k}\cdot\vec{v} = kv$

$\lambda f = v$

$\vec{s}_0 = \frac{\vec{k}}{k}$ $\qquad \vec{k} = \frac{2\pi}{\lambda}\vec{s}_0$ $\qquad \vec{s}_0\cdot\vec{r} = $ konst. → tvoří rovinu

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\delta)}$$ → cplx vyjádření

Kulová vlna

→ vznik napr. oscilácciou dipólu : $p(t) = p_0 e^{-i\omega t}$

$$P_{tot} = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$ → vyžarovaný výkon dipólom

$$\nabla^2 \psi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi(r)}{\partial r}\right)$$ → pre sféricky symetrickú funkciu $\psi(r)$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi(r)}{\partial r}\right) = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$$ → substitúcia $\psi = \frac{1}{r}\varphi$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(-\varphi + r\frac{\partial \varphi}{\partial r}\right) = \frac{1}{c^2}\frac{1}{r}\frac{\partial^2 \varphi}{\partial t^2}$$

$$\cancel{\frac{1}{r^2}}\left(-\cancel{\frac{\partial \varphi}{\partial r}} + \cancel{\frac{\partial \varphi}{\partial r}} + \cancel{r}\frac{\partial^2 \varphi}{\partial r^2}\right) = \frac{1}{c^2}\cancel{\frac{1}{r}}\frac{\partial^2 \varphi}{\partial t^2} \rightarrow \frac{\partial^2 \varphi}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2}$$

$$\Rightarrow \varphi = A e^{i(kr \pm \omega t)} \rightarrow \psi = \frac{A}{r} e^{i(kr \pm \omega t)}$$

Vlnová rovnice v materiálovom prostredí

- dominuje elektrická zložka
- nemagnetická aproximace $\underbrace{\vec{B} \approx \mu_0 \vec{H}}$

$$\nabla \times \vec{H} = \vec{j}_f + \frac{\partial \vec{D}}{\partial t} \longrightarrow \nabla \times \vec{B} = \mu_0 \vec{j}_f + \mu_0 \frac{\partial \vec{D}}{\partial t}$$, kde $\vec{D} = \varepsilon \vec{E} + \vec{P}$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = \frac{1}{\varepsilon_0}\nabla(\varrho_f + \varrho_p) - \nabla^2 \vec{E} = \frac{\nabla \varrho_f}{\varepsilon_0} + \frac{\nabla \varrho_p}{\varepsilon_0} - \nabla^2 \vec{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \frac{\partial}{\partial t}\vec{j}_f - \mu_0 \frac{\partial^2 \vec{D}}{\partial t^2} = -\mu_0 \frac{\partial \vec{j}_f}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

$$\Rightarrow \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{j}_f}{\partial t} + \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} + \frac{\nabla \varrho_f}{\varepsilon_0} + \frac{\nabla \varrho_p}{\varepsilon_0}$$ → $\varrho_p = -\nabla \cdot \vec{P}$

$$\Rightarrow \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{j}_f}{\partial t} + \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} + \frac{\nabla \varrho_f}{\varepsilon_0} - \frac{1}{\varepsilon_0}\nabla(\nabla \cdot \vec{P})$$

- nevodivé: $\rho_f = 0$, $\vec{j}_f = 0$
- izotropné: $\nabla(\nabla\cdot\vec{P}) = 0$
- nemagnetické: $\vec{B} = \mu_0 \vec{H}$ $\quad (\vec{M} = 0)$
- lineárni: $\vec{P} = \varepsilon_0 \chi(\omega)\vec{E} = \varepsilon_0(\varepsilon_r(\omega) - 1)\vec{E}$

$$\Rightarrow \nabla^2\vec{E} - \cancel{\mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} = \mu_0 \frac{\partial^2}{\partial t^2}\left(\varepsilon_0\varepsilon_r(\omega)\vec{E}\right) - \cancel{\mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\nabla^2\vec{E} = \underbrace{\mu_0\varepsilon_0\varepsilon_r(\omega)}_{\frac{1}{v^2(\omega)}} \frac{\partial^2 \vec{E}}{\partial t^2}$$

Rychlost a smér šírenia

- fázová rýchlost šírenia: $v(\omega) = \dfrac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r(\omega)}} = \dfrac{c}{\sqrt{\varepsilon_r(\omega)}} = \dfrac{c}{n(\omega)}$

$$n(\omega) = \sqrt{\varepsilon_r(\omega)} = \frac{c}{v(\omega)} \geq 1 \qquad k(\omega) = \frac{n(\omega)\,\omega}{c} \qquad \lambda(\omega) = \frac{\lambda_0(\omega)}{n(\omega)}$$

$\vec{k}\cdot\vec{r} - \omega t = k(\vec{s}_0\cdot\vec{r} - vt)$, označme $\xi = \vec{s}_0\cdot\vec{r} - vt$

$\rightarrow$ potom $\dfrac{\partial}{\partial x^i} = \dfrac{d\xi}{dx^i}\dfrac{\partial}{\partial\xi} = s_{0i}\dfrac{\partial}{\partial\xi}$ $\qquad \dfrac{\partial}{\partial t} = \dfrac{d\xi}{dt}\dfrac{\partial}{\partial\xi} = -v\dfrac{\partial}{\partial\xi}$

$$\Rightarrow \nabla\times\vec{E} = \varepsilon_{ijk}\frac{\partial}{\partial x^j}E_k = \varepsilon_{ijk}s_{0j}\frac{\partial}{\partial\xi}E_k = \vec{s}_0\times\frac{\partial\vec{E}}{\partial\xi} = -\frac{\partial\vec{B}}{\partial t} = v\frac{\partial\vec{B}}{\partial\xi}$$

$$\Rightarrow \frac{\partial}{\partial\xi}(\vec{s}_0\times\vec{E}) = \frac{\partial}{\partial\xi}(v\vec{B}) \quad\Rightarrow\quad \vec{s}_0\times\vec{E} = v\vec{B}$$

$\leftarrow \vec{E}\perp\vec{s}_0$

$$\vec{s}_0\times\vec{s}_0\times\vec{E} = \vec{s}_0\underbrace{(\vec{s}_0\cdot\vec{E})}_{0} - \vec{E}\underbrace{(\vec{s}_0\cdot\vec{s}_0)}_{1} = v\,\vec{s}_0\times\vec{B}$$

$$\Rightarrow \vec{s}_0\times\vec{B} = -\frac{1}{v}\vec{E}$$

Energia vlny

$u_E = \frac{1}{2} \vec{E} \cdot \vec{D} = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 \qquad u_B = \frac{1}{2} \vec{H} \cdot \vec{B} = \frac{1}{2\mu_0} B^2$

$B = \frac{1}{v} E \Rightarrow B^2 = \varepsilon_0 \mu_0 \varepsilon_r E^2 \Rightarrow u_B = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$

$\Rightarrow \quad u = u_E + u_B = \varepsilon_0 \varepsilon_r E^2 = \varepsilon_0 \varepsilon_r v \vec{E} \cdot \vec{B}$

↳ pre vlnu v tvare $\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega z)$ je $\langle u_E \rangle = \frac{1}{4} \varepsilon_0 \varepsilon_r E_0^2$

→ celková hustota energie potom:

$$U = \langle u_E \rangle + \langle u_B \rangle = \frac{\varepsilon_0 \varepsilon_r}{2} E_0^2$$

→ žiarivý výkon $\langle c u \rangle = \langle \varepsilon_0 E^2 c \rangle$

Poyntingov vektor:

$$\vec{S} = \vec{E} \times H = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

E je kolmé na $B \Rightarrow S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0 v} E^2$

$$\Rightarrow \quad \langle S \rangle = \frac{1}{2} \frac{E_0^2}{\mu_0 v} = \frac{1}{2} \frac{E_0^2}{\mu_0} \frac{n}{c} = \frac{1}{2} \varepsilon_0 c n E_0^2$$

→ definujeme intenzitu vlny: $I := \langle S \rangle = \frac{1}{2} \varepsilon_0 c n E_0^2$

Polarizace monochromatické vlny

Polarizačná elipsa

- zvolíme vlnu v tvare

$$E_x = a_x e^{i(kz-\omega t)} = a_x \cos(kz-\omega t)$$
$$E_y = a_y e^{i(kz-\omega t-\delta)} = a_y \cos(kz-\omega t-\delta)$$

} eliptický valec

→ v závislosti na a_x, a_y, δ dostaneme rôzne druhy polarizácií:

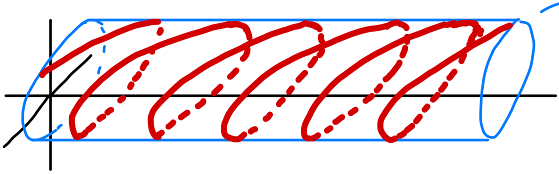
$$E_y = a_y \cos(kz-\omega t)\cos\delta + a_y \sin(kz-\omega t)\sin\delta$$

$$\frac{E_y}{a_y} = \frac{E_x}{a_x}\cos\delta + \sqrt{1-\left(\frac{E_x}{a_x}\right)^2}\sin\delta$$

$$\left(\frac{E_y}{a_y}\right)^2 + \left(\frac{E_x}{a_x}\right)^2\cos^2\delta - 2\frac{E_y}{a_y}\frac{E_x}{a_x}\cos\delta = \sin^2\delta - \sin^2\delta\left(\frac{E_x}{a_x}\right)^2$$

$$\left(\frac{E_y}{a_y}\right)^2 - 2\frac{E_y}{a_y}\frac{E_x}{a_x}\cos\delta + \left(\frac{E_x}{a_x}\right)^2 = \sin^2\delta$$

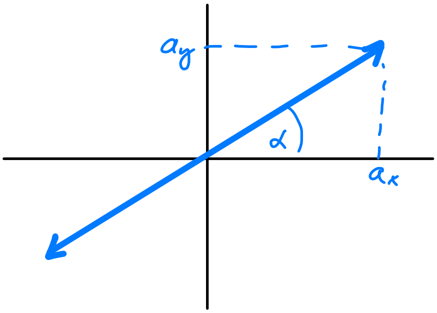
- $\delta = \pm\frac{\pi}{2}$, $a_x = a_y$ → kruhovo polarizované
- $\delta = 0$ → lin. polarizované
- obecne elipsa



Jonesov formalizmus

- polarizačný stav = vektor
- optické prvky meniace polarizáciu = matice

→ platí len pre úplne polarizované svetlo



$$\sin\alpha = \frac{a_y}{\sqrt{a_x^2+a_y^2}} \qquad \cos\alpha = \frac{a_x}{\sqrt{a_x^2+a_y^2}}$$

$$E_x = a_x e^{i\varphi} = \sqrt{a_x^2+a_y^2}\,\cos\alpha\, e^{i\varphi}$$
$$E_y = a_y e^{i(\varphi-\delta)} = \sqrt{a_x^2+a_y^2}\,\sin\alpha\, e^{-i\delta} e^{i\varphi}$$

$$E_{eff} := \sqrt{a_x^2+a_y^2}$$

$$\vec{E} = E_{ef}\underbrace{\begin{pmatrix}\cos\alpha \\ \sin\alpha\, e^{-i\delta}\end{pmatrix}}_{J = \begin{pmatrix}\cos\alpha \\ \sin\alpha\, e^{-i\delta}\end{pmatrix}} e^{i\varphi} = E_{ef}\,\vec{J}\,e^{i\varphi}$$

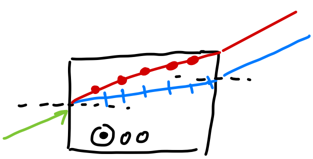
$J = \begin{pmatrix}\cos\alpha \\ \sin\alpha\, e^{-i\delta}\end{pmatrix}$ → Jonesov vektor, $|\vec{J}| = 1$

$$\langle w_E \rangle = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E_{ef}^2$$

- špeciálne prípady:
  $J = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ -i\end{pmatrix}$ → RCP
  $J = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ i\end{pmatrix}$ → LCP

Príprava polarizovaného stavu

- lineárni dvojlom



– rôzne n v rôznych smeroch

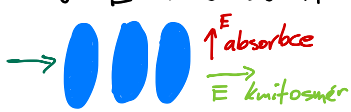
- odraz

od povrchu, pre $tg\,\Theta_{Br} = \frac{n_2}{n_1}$

Brew. uhol

- dichroismus

– v jednom smere je $\vec{E}$ absorbované



Polarizačné zariadenia

- rotátor

$$T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

- polarizátor

→ projekce na vektor $\vec{p} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$

⇒ T je projekčná matica:

$$T = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \begin{pmatrix} \cos\beta & \sin\beta \end{pmatrix} = \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}$$

↳ nech $\vec{p} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ a $E_y = E_0 \cos\gamma \, e^{i\varphi}$ je polarizované svetlo, kde γ je smer, meraný od $\vec{p}$

- $I = \frac{1}{2} \varepsilon_0 n c E_0^2 \cos^2\gamma = I_0 \cos^2\gamma$

→ ak máme nepolarizované svetlo a $\gamma \in [0, 2\pi]$, tak pre celk. intenzitu platí:

$$I = \frac{1}{2\pi} \int_0^{2\pi} I_0 \cos^2\gamma \, d\gamma = \frac{I_0}{2\pi} \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2\gamma \right) d\gamma = \frac{I_0}{2}$$

⇒ pre obecne nepolarizované svetlo polarizátor prepustí polovicu I

- fázová doštička

$$\phi = \frac{2\pi}{\lambda}(n_y - n_x) d \quad \rightarrow \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

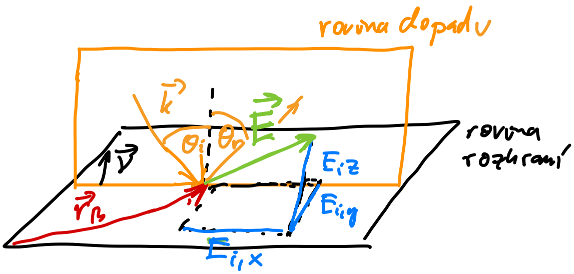
Odraz a lom

Zákon odrazu a lomu

- z Maxwellových rovnic plyne:

(1) $\vec{E}_{i,\,tečná} + \vec{E}_{r,\,tečná} = \vec{E}_{t,\,tečná}$

(2) $\vec{H}_{i,\,tečná} + \vec{H}_{r,\,tečná} = \vec{H}_{t,\,tečná}$



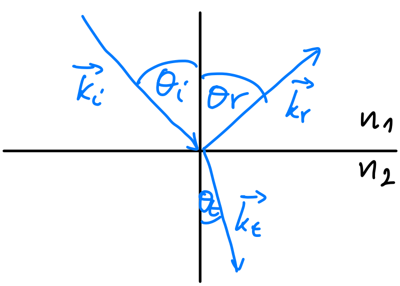
→ z (1) dostaneme: $\vec{k}_i \cdot \vec{r} - \omega_i t = \vec{k}_r \cdot \vec{r} - \omega_r t = \vec{k}_t \cdot \vec{r} - \omega_t t$

- porovnaním časových koef. dostaneme: $\omega_i = \omega_r = \omega_t$
- porovnaním poloh koef. — // — : $\vec{k}_i \cdot \vec{r}_B = \vec{k}_r \cdot \vec{r}_B = \vec{k}_t \cdot \vec{r}_B$

→ $\vec{r}_B$ je vektor v xy rovine: $\vec{r}_B = (x, y, 0)$

→ $\vec{k}_i = (k_{ix}, 0, 0)$ ⇒ $k_{ix} x = k_{rx} x + k_{ry} y = k_{tx} x + k_{ty} y$

⇒ porovnaním koef dostaneme $k_{ry} = k_{ty} = 0$ ⇒ $\vec{k}_i, \vec{k}_r, \vec{k}_t$ v 1 rovine



$k_{ix} = k_{rx}$ ⇒ $\frac{2\pi}{\lambda} n_1 \sin\theta_i = \frac{2\pi}{\lambda} n_1 \sin\theta_r$

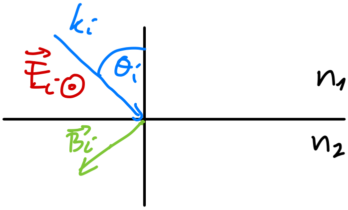
⇒ $\sin\theta_i = \sin\theta_r$ ⇒ $\theta_i = \theta_r$

$k_{ix} = k_{tx}$ ⇒ $\frac{2\pi}{\lambda} n_1 \sin\theta_i = \frac{2\pi}{\lambda} n_2 \sin\theta_t$

⇒ $n_1 \sin\theta_i = n_2 \sin\theta_t$

Fresnelove vzorce

- s-polarizace = kolmo na rovinu dopadu: $\vec{E}_i = (0, E_{iy}, 0)$



$$\vec{k}_i = (k_{ix}, 0, k_{iz}) = k(\sin\theta_i, 0, \cos\theta_i)$$

platí: $\vec{s}_0 \times \vec{E}_i = v\vec{B}_i = \frac{c}{n_1}\vec{B}_i$, kde $\vec{s}_0 = \frac{\vec{k}_i}{k} = (\sin\theta_i, 0, \cos\theta_i)$

$$\Rightarrow \quad \vec{B}_i = \frac{n_1}{c}\vec{s}_0 \times \vec{E}_i = \frac{n_1}{c}E_{iy}(-\cos\theta_i, 0, \sin\theta_i)$$

$$\vec{B}_t = \frac{n_2}{c}\vec{s}_0 \times \vec{E}_t = \frac{n_2}{c}E_{ty}(-\cos\theta_t, 0, \sin\theta_t)$$

$$\vec{B}_r = \frac{n_1}{c}\vec{s}_0 \times \vec{E}_r = \frac{n_1}{c}E_{ry}(\cos\theta_i, 0, \sin\theta_i)$$

- ze spojitosti: $B_{ix} + B_{rx} = B_{tx}$ $\qquad E_{iy} + E_{ry} = E_{ty}$

$$\Rightarrow \quad -\cos\theta_i n_1 E_{iy} + \cos\theta_i n_1 E_{ry} = -n_2 E_{ty}\cos\theta_t = -n_2(E_{iy} + E_{ry})\cos\theta_t$$

$$-\cos\theta_i n_1 + \cos\theta_i n_1 \frac{E_{ry}}{E_{iy}} = -n_2\cos\theta_t - n_2\frac{E_{ry}}{E_{iy}}\cos\theta_t$$

definujeme: $$r_s = \frac{E_{ry}}{E_{iy}} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$t_s = \frac{E_{ty}}{E_{iy}} = r_s + 1$$

- p-polarizace

$$r_p = \frac{E_r}{E_i} = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} = \frac{\operatorname{tg}(\theta_t - \theta_i)}{\operatorname{tg}(\theta_t + \theta_i)}$$

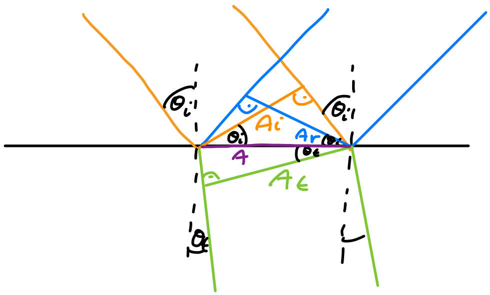
$$t_s = (r_s + 1)\frac{\cos\theta_i}{\cos\theta_t}$$

Výkonové koeficienty a energetická bilance

• intenzity jednotlivých vĺn sú:

$$I_i = \frac{1}{2}\varepsilon_0 c n_1 E_{0i}^2$$
$$I_t = \frac{1}{2}\varepsilon_0 c n_2 E_{0t}^2$$
$$I_r = \frac{1}{2}\varepsilon_0 c n_1 E_{0r}^2$$

• výkon J je daný ako $J = I \cdot A$, kde A je priečny prierez vlny



$$A = \frac{A_i}{\cos\Theta_i} = \frac{A_r}{\cos\Theta_i} = \frac{A_t}{\cos\Theta_t}$$

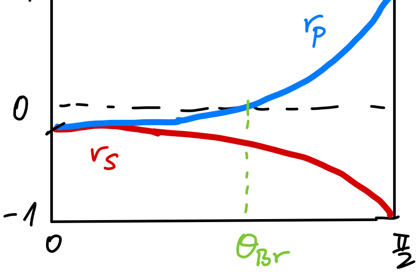
$$\Rightarrow J_i = \frac{1}{2}\varepsilon_0 c n_1 E_{0i}^2 A\cos\Theta_i$$
$$J_r = \frac{1}{2}\varepsilon_0 c n_1 E_{0r}^2 A\cos\Theta_i$$
$$J_t = \frac{1}{2}\varepsilon_0 c n_2 E_{0t}^2 A\cos\Theta_t$$

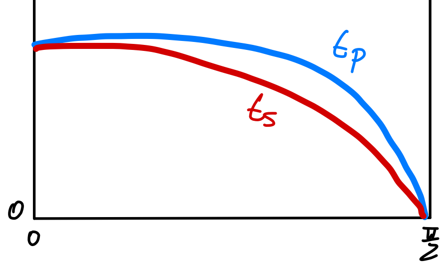
• ZZE: $J_i = J_r + J_t \Rightarrow n_1 E_{0i}^2 \cos\Theta_i = n_1 E_{0r}^2 \cos\Theta_i + n_2 E_{0t}^2 \cos\Theta_t$

• výkonové koeficienty:

$$R_{s,p} = \frac{J_r}{J_i} = \frac{E_{0r}^2}{E_{0i}^2} = |r_{s,p}|^2 \qquad T_{s,p} = \frac{J_t}{J_i} = \frac{n_2\cos\Theta_t}{n_1\cos\Theta_i}|t_{s,p}|^2$$

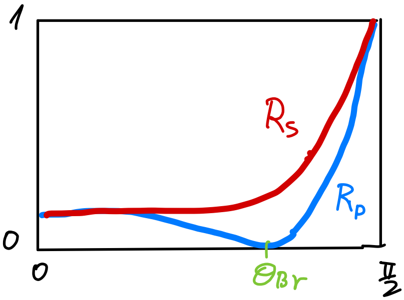
Opticky hustejšie prostredie

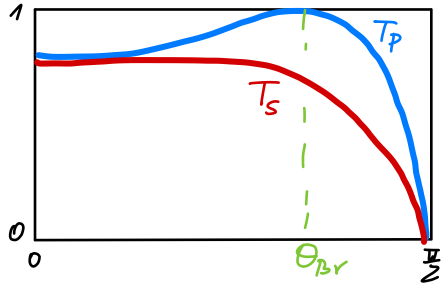




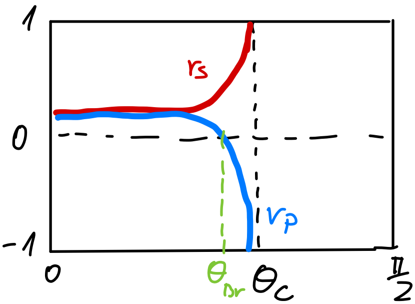
• ak $\Theta_i = \Theta_{Br}$, t.j. $\tan\Theta_i = \frac{n_2}{n_1}$, tak $r_p = 0 \Rightarrow$ plne kolmo polarizované

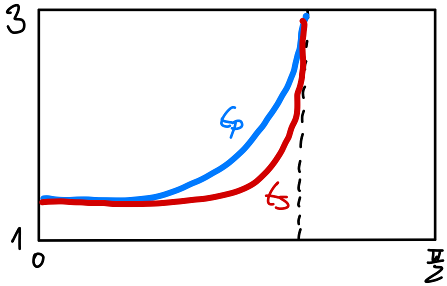
• $r_s < 0 \Rightarrow$ obracia polarizáciu



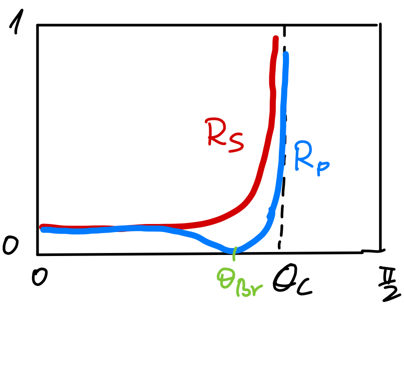


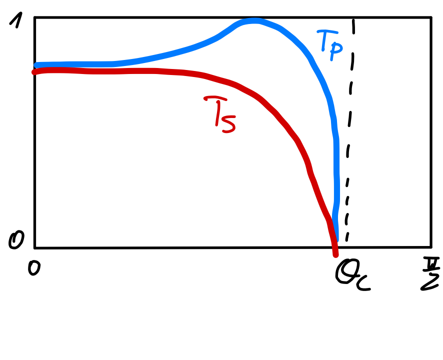
Opticky redšie prostredie





- $r_s > 0$ ⇒ neotáča polarizáciu
- pre $\theta \geq \theta_c$ nastáva totálny odraz s rôznymi fázovými posunmi
  ↳ do 2. prostredia preniká evanescentná vlna, exp. tlmená





Interference 1

monochromatické vlny

− 2 spôsoby − delenie vlnoplochy
− delenie amplitúdy − beamsplitter

Dvojzväzková interferencia rovinných vĺn

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k}_1 \cdot \vec{r} - \omega t + \delta_{01})} \qquad |\vec{k}_1| = |\vec{k}_2|$$
$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t + \delta_{02})}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = \frac{1}{4}\varepsilon_0 n^2 \vec{E} \cdot \vec{E}^* = \frac{1}{4}\varepsilon_0 n^2 \left(E_{01}^2 + E_{02}^2 + \vec{E}_{01} \cdot \vec{E}_{02} e^{i(\varphi_1 - \varphi_2)} + \vec{E}_{01} \cdot \vec{E}_{02} e^{-i(\varphi_1 - \varphi_2)}\right)$$

$$= I_1 + I_2 + \frac{1}{4}\varepsilon_0 n^2 \, 2E_{01}E_{02} \cos\alpha \cos(\varphi_1 - \varphi_2) =$$
$$= I_1 + I_2 + \frac{1}{4}\varepsilon_0 n^2 \, 2E_{01}E_{02} \cos\alpha \cos\delta_{12}(\vec{r})$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\alpha \cos\delta_{12} \qquad \delta_{12} = k(\vec{s}_{01} - \vec{s}_{02}) \cdot \vec{r} + \delta_{01} - \delta_{02}$$

pre $\underline{\cos\alpha = 1}$:

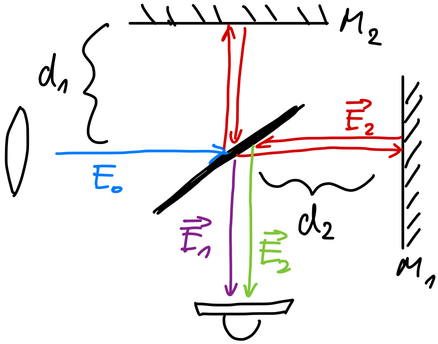
$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad \delta_{12} = 0$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \qquad \delta_{12} = \pi$$

viditeľnosť

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

nech $I_1 = I_2 = I_0$, potom $I = 2I_0(1 + \cos\delta_{12}) = 4I_0 \cos^2\frac{\delta_{12}}{2}$

Michelsonov interferometer $(\vec{k}_1 \parallel \vec{k}_2)$



$$\delta_{12} = \varphi_2 - \varphi_1 = k2(d_2 - d_1) = \frac{4\pi n}{\lambda}(d_2 - d_1)$$

→ maximum: $\delta_{12} = 2m\pi$:

$$d_2 - d_1 = \frac{m\lambda}{2n}$$

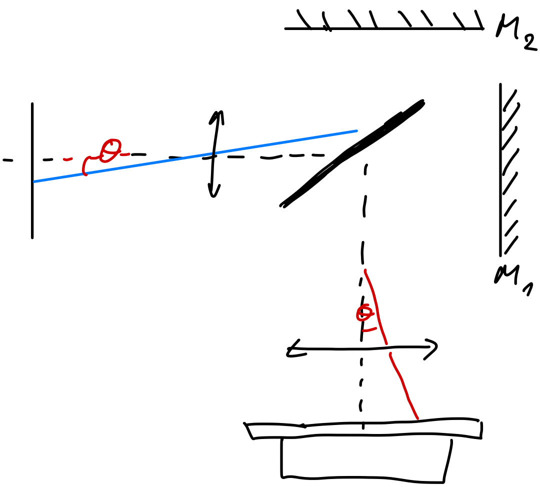
→ minimum: $\delta_{12} = \pi(2m+1)$

$$d_2 - d_1 = \frac{(m+\frac{1}{2})\lambda}{2n}$$

$$\left.\begin{aligned} E_1 &= t_{BS} r_{M_1} r_{BS} E_0 \\ E_2 &= r_{BS} r_{M_2} t_{BS} E_0 \end{aligned}\right\}$$ ak budú zrkadlá rovnaké, tak $E_1 = E_2$, teda $I_1 = I_2 = I_0$

$$\Rightarrow \quad I = 4 I_0 \cos^2\left(\frac{2\pi n}{\lambda}(d_2 - d_1)\right)$$ → nezávisí na polohe

• zmeníme uhol strielaného paprsku :



$$\delta_{12} = k2(d_1 - d_2)\cos\theta$$
$$= \frac{2\pi}{\lambda} 2n(d_1 - d_2)\cos\theta$$

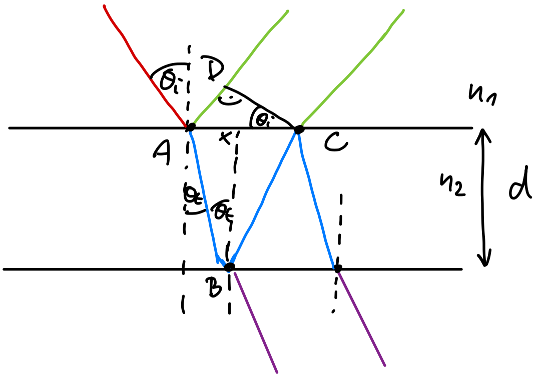
• maximá:

$$(d_1 - d_2)\cos\theta = \frac{\lambda m}{2n}$$

• minimá

$$(d_1 - d_2)\cos\theta = \frac{\lambda(m+\frac{1}{2})}{2n}$$

Planparalelní destička $(\vec{k}_1 \parallel \vec{k}_2)$



$$\sin\theta_i = \frac{|AD|}{x}$$

$$|ABC| = 2|AB| = \frac{x}{\sin\theta_t}$$

$$tg\,\theta_t = \frac{x/2}{d} \Rightarrow x = 2 tg\,\theta_t d$$

$$\delta = \frac{2\pi}{\lambda} n_2 |ABC| - \frac{2\pi}{\lambda} n_1 |AD| = \frac{2\pi}{\lambda} n_2 \frac{x}{\sin\theta_t} - \frac{2\pi}{\lambda} n_1 x \sin\theta_i =$$

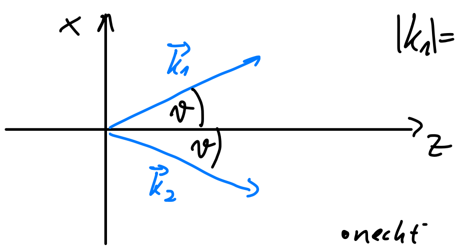
$$= \frac{2\pi}{\lambda} n_2 \left(\frac{2 tg\,\theta_t d}{\sin\theta_t} - 2 tg\,\theta_t\, d \sin\theta_t \right) = \frac{2\pi}{\lambda} 2 n_2 d \cos\theta_t$$

$$\delta_{\bullet} = \frac{4\pi}{\lambda} n_2 d \cos\theta_t$$

→ pre prípad • ak $n_1 < n_2$ je nutné započítať fázový posun pri odraze:

$$\delta_{\bullet} = \frac{4\pi}{\lambda} n_2 d \cos\theta_t + \pi$$

Nekolineárne vlnové vektory



$$|k_1| = |k_2| = k$$

$$\vec{k}_1 = k(\sin\vartheta, 0, \cos\vartheta) = (k_x, 0, k_z)$$

$$\vec{k}_2 = k(-\sin\vartheta, 0, \cos\vartheta) = (-k_x, 0, k_z)$$

nech je E_1, E_2 v smere y

$$E_{y1} = E_0 e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)} = E_0 e^{i(k_z z - \omega t)} e^{i(k_x x + \frac{\delta_0}{2})}$$

$$E_{y2} = E_0 e^{i(\vec{k_2}\cdot\vec{r} - \omega t)} = E_0 e^{i(k_z z - \omega t)} e^{-i(k_x x + \frac{\delta_0}{2})}$$

$$E_y = E_{1y} + E_{2y} = E_0 e^{i(k_z z - \omega t)} \left(e^{i(k_x x + \frac{\delta_0}{2})} + e^{-i(k_x x + \frac{\delta_0}{2})} \right) =$$

$$= E_0 e^{i(k_z z - \omega t)} \, 2\cos\left(k_x x + \frac{\delta_0}{2}\right)$$

$$I = \frac{1}{2}\varepsilon_0 n^2 E_0^2 \, 4 . \cos^2\left(k_x x + \frac{\delta_0}{2}\right) = 4 I_0 \left(1 + \cos\left(2k_x x + \delta_0\right)\right) =$$

$$= 4 I_0 \left(1 + \cos\left(\frac{4\pi}{\lambda_0} \sin\vartheta \, x + \delta_0\right)\right)$$

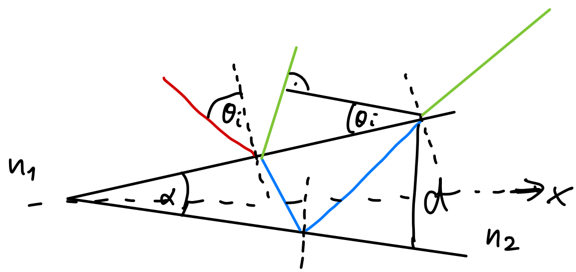
- maximum:

$$\frac{4\pi}{\lambda_0} \sin\vartheta \, x_{max} + \delta_0 = 2m\pi$$

- minimum

$$\frac{4\pi}{\lambda_0} \sin\vartheta \, x_{min} + \delta_0 = (2m+1)\pi$$

Interferenční proužky



- použijeme aproximáciu

$$tg\,\alpha \approx \alpha = \frac{d}{x}$$

→ potom môžeme použiť vzťah pre plan paralelní destičku:

$$2nd\cos\theta_t = \lambda m \quad \rightarrow \min \quad \Rightarrow x_{min} = \frac{\lambda m}{2n\alpha}\cos\theta_t$$

$$2nd\cos\theta_t = \lambda\left(m + \frac{1}{2}\right) \quad \rightarrow \max \quad \Rightarrow x_{max} = \frac{\lambda\left(m + \frac{1}{2}\right)}{2n\alpha}\cos\theta_t$$

Interference mnoha vln

• stejné amplitudy

$$E_m = E_0 e^{i\varphi} e^{im\delta}$$

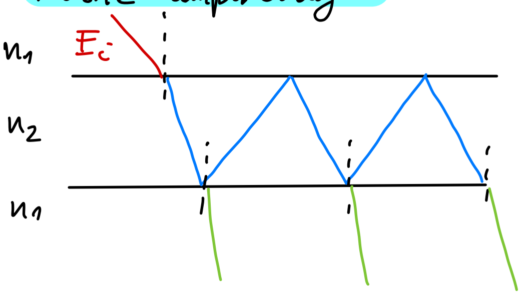
$$E_{tot} = E_0 e^{i\varphi} \sum_{m=1}^{N} e^{im\delta} = E_0 e^{i\varphi} \frac{1-e^{iN\delta}}{1-e^{i\delta}} =$$

$$|E_{tot}|^2 = E_0^2 \frac{1-e^{iN\delta}}{1-e^{i\delta}} \cdot \frac{1-e^{-iN\delta}}{1-e^{-i\delta}} = E_0^2 \frac{2-2\cos N\delta}{2-2\cos\delta} =$$

$$= E_0^2 \frac{1-\cos N\delta}{1-\cos\delta} = E_0^2 \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$$

$$\Rightarrow I = N^2 I_0 \frac{\sin^2\left(\frac{N\delta}{2}\right)}{N^2 \sin^2\frac{\delta}{2}}$$

• různé amplitudy



δ je dané prejdenou vzdalenostou

$$E_{0,t} = t_{12} t_{21} e^{i\frac{\delta}{2}} E_i$$

$$E_{1,t} = t_{21} r_{21} r_{21} t_{12} e^{i\frac{\delta}{2}} e^{i\delta} E_i$$

$$E_{2,t} = t_{21} (r_{21} r_{21})^2 t_{12} e^{i\frac{\delta}{2}} e^{i2\delta} E_i$$

$$\vdots$$

$$E_{l,t} = t_{21} r_{21}^{2l} t_{12} e^{i\frac{\delta}{2}} e^{il\delta} E_i$$

$$E_{tot} = \sum_{l=1}^{\infty} E_{l,t} = t_{21} t_{12} e^{i\frac{\delta}{2}} E_i \sum_{l=1}^{\infty} \left(r_{21}^2 e^{i\delta}\right)^l =$$

$$= t_{21} t_{12} e^{i\frac{\delta}{2}} E_i \frac{1}{1-r_{21}^2 e^{i\delta}}$$

$r_{12} = -r_{21} = r$

$t_{21} t_{12} = (1+r_{12})(1+r_{21})$

$$I = \frac{1}{4}\varepsilon_0 n_1^2 E_{tot} E_{tot}^* = \frac{1}{4}\varepsilon_0 n_1^2 E_0^2 \overbrace{(t_{21} t_{12})^2}^{(1-r^2)^2} \frac{1}{1+r^4-2r^2\cos\delta}$$

$$= I_0 \frac{(1-r^2)^2}{(1-r^2)^2+2r^2(1-\cos\delta)} = I_0 \frac{1}{1+\frac{2r^2}{(1-r^2)^2}(1-\cos\delta)} = I_0 \frac{1}{1+\frac{4r^2}{(1-r^2)^2}\sin^2\frac{\delta}{2}}$$

$$= \frac{I_0}{1+F\sin^2\frac{\delta}{2}}$$

$F = \frac{4r^2}{(1-r^2)^2} = \frac{4R}{(1-R)^2}$ ... jemnost

Interference 2

polychromatické vlny

Dve vlny rôznej frekvencie

$$E_1 = E_0 \cos(k_1 z - \omega_1 t)$$
$$E_2 = E_0 \cos(k_2 z - \omega_2 t)$$

$$E = E_1 + E_2 = E_0\big(\cos(k_1 z - \omega_1 t) + \cos(k_1 z - \omega_1 t)\big) =$$
$$= E_0 \cos\left(\frac{k_1+k_2}{2} z - \frac{\omega_1+\omega_2}{2} t\right) \cos\left(\frac{k_1-k_2}{2} z - \frac{\omega_1-\omega_2}{2} t\right)$$

$$\bar{\omega} = \frac{\omega_1+\omega_2}{2} \qquad \bar{k} = \frac{k_1+k_2}{2}$$

$$\delta k = \frac{k_1-k_2}{2} = \frac{\Delta k}{2} \qquad \delta\omega = \frac{k_1-k_2}{2} = \frac{\Delta\omega}{2}$$

$$E = E_0 \cos(\bar{k}z - \bar{\omega}t) \cos(\delta k\, z - \delta\omega\, t)$$

- pre $\delta k \ll \bar{k}$, $\delta\omega \ll \bar{\omega}$ je druhá časť modulačná

Grupová a fázová rýchlosť

$$v_g = \frac{\delta\omega}{\delta k} \rightarrow \frac{d\omega}{dk}\Big|_{\bar{k}} \quad \text{– grupová rýchlosť}$$

$$v_f = \frac{\bar{\omega}}{\bar{k}} \quad \text{– fázová rýchlosť}$$

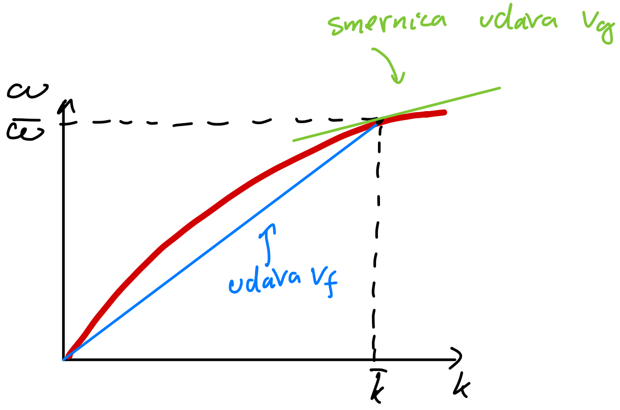
- pre vákuum: $c = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} \Rightarrow v_g = \frac{\omega_1-\omega_2}{k_1-k_2} = c\,\frac{\omega_1-\omega_2}{\omega_1-\omega_2} = c = v_f$

- disperzné prostredie

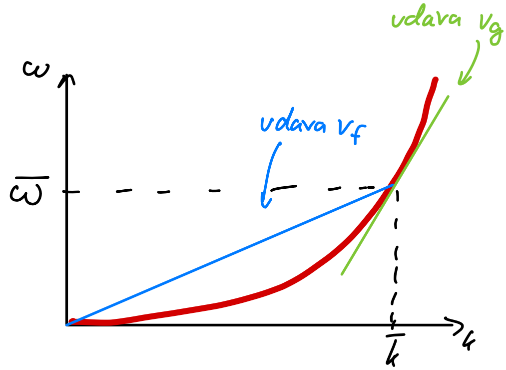
$$k(\omega) = \frac{\omega}{c} n(\omega)$$

$$v_g = \frac{d\omega}{dk} = \left(\frac{dk}{d\omega}\right)^{-1} = \left(\frac{n(\omega)}{c} + \frac{\omega n'(\omega)}{c}\right)^{-1} = \frac{\frac{c}{n(\omega)}}{1 + \omega\frac{n'(\omega)}{n(\omega)}}$$

$$V_g = \frac{V_f}{1 + \omega \frac{n'(\omega)}{n(\omega)}}$$



$V_f > V_g$ → normálna disperzia



$V_f < V_g$ → abnormálna dis.

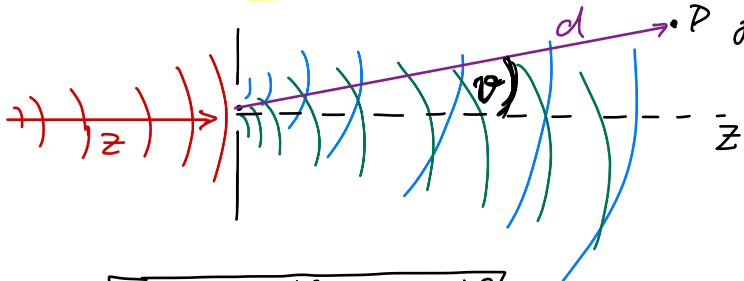
Difrakce

- najjednoduchší prípad – skalárna aproximácia

Difrakčný integrál

$$E(x,y,z) = -\frac{i}{\lambda}\int E(X,Y,0)\,\frac{e^{ikd}}{d}\,K(\vartheta)\,dX\,dY$$

aparatura



$$d = \sqrt{z^2 + (x-X)^2 + (y-Y)^2} \rightarrow \text{vzdialenosť}$$

$K(\vartheta)$ je uhlový faktor, budeme predpokladať $K(\vartheta) \approx 1$

Fresnelova aproximace

$$d = z\sqrt{1 + \frac{(x-X)^2 + (y-Y)^2}{z^2}} \approx z + \frac{(x-X)^2 + (y-Y)^2}{2z}$$

→ predpokladáme veľké vzdialenosti od apertúry, teda

$$(x-X)^2 + (y-Y)^2 \ll z^2$$

$$E(x,y,z) = \frac{-i}{\lambda}\int E(X,Y,0)\,\frac{e^{ikz}}{z}\,e^{ik\frac{(x-X)^2}{2z}}\,e^{ik\frac{(y-Y)^2}{2z}}\,dX\,dY$$

$$= -\frac{i}{\lambda}\,\frac{e^{ikz}}{z}\,e^{\frac{ik(x^2+y^2)}{2z}}\int E(X,Y,0)\,e^{\frac{ik(X^2+Y^2)}{2z}}\,e^{-ik\frac{(xX+yY)}{z}}\,dX\,dY$$

- malé otvory
- veľká vzdialenosť

Fraunhofova aproximace

• zanedbanie člena $e^{ik\frac{(X^2+Y^2)}{2z}}$:

$$E(x,y,z) = -\frac{i}{\lambda}\,\frac{e^{ikz}}{z}\,e^{ik\frac{(x^2+y^2)}{2z}}\int E(X,Y,0)\,e^{-ik\frac{(xX+yY)}{z}}\,dX\,dY$$

→ platí pre $e^{ik\frac{(X^2+Y^2)}{2z}} \approx 1 \quad \Rightarrow \quad 2z \gg k(X^2+Y^2)$

• otvor aproximujeme kruhom $X^2+Y^2=\left(\frac{D}{2}\right)^2$, D je max. rozmer otvoru:

$$z \gg z_{MEZ} \approx k\frac{D^2}{8}$$

Intenzita na priamke kruhového otvoru

$$E(0,0,z) = -\frac{i}{\lambda}\int_{kruh} E_0\,\frac{e^{ikr}}{r}\,dS = -\frac{i}{\lambda}E_0\int \frac{e^{ik\sqrt{X^2+Y^2+z^2}}}{\sqrt{X^2+Y^2+z^2}}\,dX\,dZ$$

$$= \left| X=\rho\sin\varphi \quad Y=\rho\cos\varphi \right| = -\frac{i}{\lambda}E_0\int_0^{2\pi}\int_0^{D/2}\frac{e^{ik\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}}\,\rho\,d\rho\,d\varphi =$$

$$= \left| \begin{matrix} \xi=\sqrt{\rho^2+z^2} \\ d\xi=\frac{\rho}{\sqrt{\rho^2+z^2}}d\rho \end{matrix} \right| = -\frac{i}{\lambda_0}E_0\int_0^{2\pi}\int_z^{\sqrt{\frac{D^2}{4}+z^2}} e^{ik\xi}\,d\xi\,d\varphi =$$

$$= -\frac{\cancel{i}}{\cancel{\lambda_0}}E_0\,\cancel{2\pi}\left[\frac{e^{ik\xi}}{\cancel{i}\cancel{k}}\right]_z^{\sqrt{\frac{D^2}{4}+z^2}} = E_0\left(e^{ikz}-e^{ik\sqrt{\frac{D^2}{4}+z^2}}\right)$$

$$\Rightarrow I \sim |E|^2 = 2E_0^2\left(1-\cos\left(k\left[z-\sqrt{\frac{D^2}{4}+z^2}\right]\right)\right)$$

• Fraunhofova aproximace:

$$E(0,0,z) = -\frac{i}{\lambda}\,\frac{e^{ikz}}{z}\int_{kruh} E_0\,dX\,dY = -\frac{i}{\lambda}\,\frac{e^{ikz}}{z}\,E_0\,\pi\,\frac{D^2}{4} \Rightarrow I = \frac{E_0^2\pi^2D^4}{16\lambda^2z^2}$$

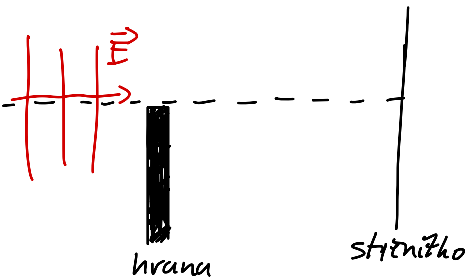
Babinetov princip

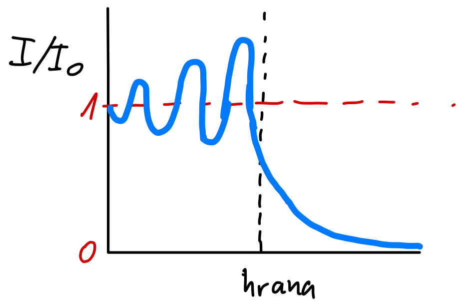
$$E_{\text{prekážka}} + E_{\text{otvor}} = E_{\text{bez prekážky}}$$

$$E_{\text{kruhová diera}} = E_{\text{bez prekážky}} - E_{\text{otvor}} = \cancel{E_0 e^{ikz}} - E_0\left(\cancel{e^{ikz}} - e^{ik\sqrt{\frac{D^2}{4}+z^2}}\right)$$
$$= E_0 e^{ik\sqrt{\frac{D^2}{4}+z^2}}$$

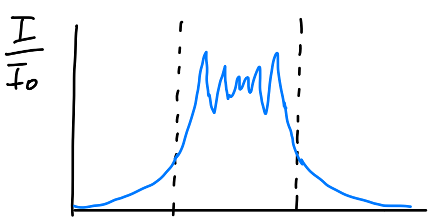
$\rightarrow I(0,0,z) = |E_{\text{kruh. diera}}|^2 = E_0^2$ $\Rightarrow$ Poissonova škvrna

Difrakce na hrane (Fresnel)

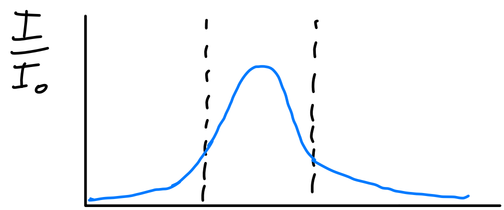




Difrakce na štrbine

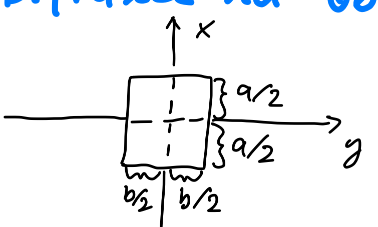


Fresnelova app.



Fraunhof. app.

Difrakce na obdelníku (Fraunhof)



$$E(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{ik\frac{(x^2+y^2)}{2z}} E_0 \int_{\text{obdelník}} e^{ik\frac{(xX+yY)}{z}} \, dXdY$$

$$= -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{ik\frac{(x^2+y^2)}{2z}} E_0 \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{\frac{ikx}{z}X} dX \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{\frac{iky}{z}Y} dY -$$

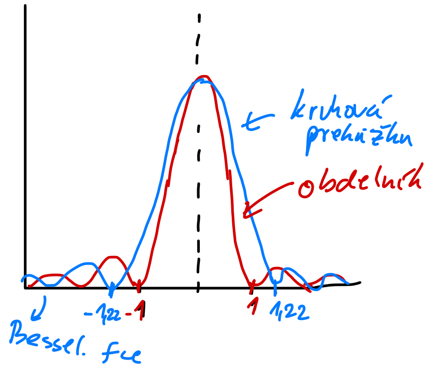
$$\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{\frac{ikx}{z}X} dX = \left[\frac{e^{\frac{ikx}{z}X}}{i\frac{kx}{z}}\right]_{-a/2}^{a/2} = \frac{z}{ikx} 2i \sin\left(\frac{akx}{2z}\right) = a\frac{\sin u}{u}$$

$$u = \frac{akx}{2z}$$

→ analogicky aj $\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{\frac{iky}{z}Y} dY = b\frac{\sin v}{v}$ $\quad v = \frac{bky}{2z}$

$$E = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{ik\frac{(x^2+y^2)}{2z}} E_0 \left(b\frac{\sin v}{v}\right)\left(a\frac{\sin u}{u}\right)$$

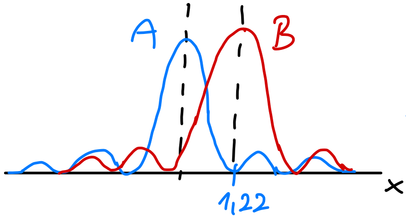
$$I \sim EE^* = \left(\frac{E_0}{\lambda z}ab\right)^2 \frac{\sin^2 v}{v^2}\frac{\sin^2 u}{u^2}$$

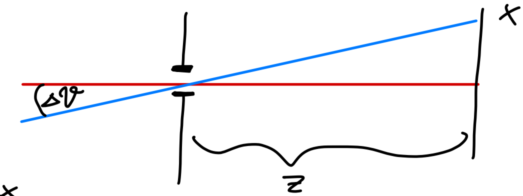


- prvé minimum
  1) obdelníkové apertúry v 1
  2) kruhové apertúry v 1,22

Rayleighovo kritérium

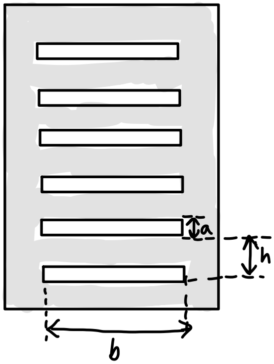
- kritérium pre schopnosť rozlíšenia dvoch bodov
- stanoví uhol, pod kt. sa dajú rozlíšiť 2 body A, B
- A, B sa dajú rozlíšiť, ak hlavné max. A je zhodné s prvým min B





$$\Delta\vartheta = \frac{x}{z} = 1{,}22\frac{\lambda}{D}$$

Amplitúdová difrakční mřížka



• napíšeme ako súčet vĺn, kde každá prechádza jedným obdĺžnikom

$$E(X,Y,0) = \sum_{n=1}^{N} E(X-X_n, Y-Y_n, 0)$$

$$E(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{ik\frac{(x^2+y^2)}{2z}} \sum_{n=1}^{N} \int E(X-X_n, Y-Y_n, 0) e^{-ik\frac{xX-yY}{z}} dX dY$$

urobením substitúcie $X' = X - X_n$, $Y' = Y - Y_n$ môžeme

$$E(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{ik\frac{(x^2+y^2)}{2z}} \sum_{n=1}^{N} e^{-ik\left(\frac{xX_n+yY_n}{z}\right)} \int E(X',Y',0) e^{-ik\frac{xX'-yY'}{z}} dX'dY'$$

→ čo môžeme napísať ako

$$E = E_0 f_1 \sum_{n=1}^{N} e^{-ik\frac{xX_n+yY_n}{z}} = E_0 f_1 N F_N$$

↳ kde f_1 je dané disperziou na obdelníku

$$X_N = \left(n - \frac{N+1}{2}\right)h \qquad Y_n = 0$$

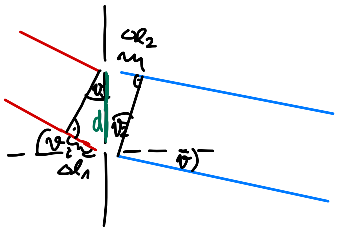
$$\Rightarrow \quad F_N = \frac{1}{N} e^{ik\frac{N+1}{2}\frac{hx}{z}} \sum e^{-ik\frac{nhx}{z}} = \frac{\sin(\eta N)}{N\sin(\eta)} \qquad \eta = \frac{hx}{2z}$$

$$\Rightarrow \quad I(x,0,z) \propto I_0 \left(\frac{\sin N\eta}{N\sin\eta}\right)^2 \left(\frac{\sin v}{v}\right)^2 \left(\frac{\sin u}{u}\right)^2$$

$$u = \frac{ak}{2}\sin\vartheta \qquad v = \frac{bk}{2}\sin\varphi$$

ϑ je uhol dopadu a φ uhol pozorovania

Mriežková rovnica

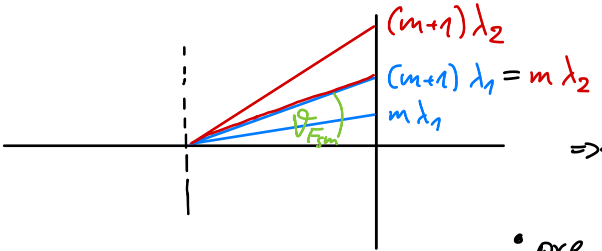


$$\delta = k(\Delta l_2 - \Delta l_1) = \frac{2\pi}{\lambda}(d\sin\vartheta - d\sin\vartheta_i)$$

- maximá: $m\lambda = d(\sin\vartheta - \sin\vartheta_i)$
- minimá: $(m+\frac{1}{2})\lambda = d(\sin\vartheta - \sin\vartheta_i)$

- uhlová disperze: $D_\vartheta = \left.\frac{d\vartheta}{d\lambda}\right|_{\vartheta_i = konst} = \frac{m}{d\cos\vartheta}$

- voľný spektrálny interval – interval vlnových dĺžok $[\lambda_1, \lambda_2]$, pre kt. platí, že $\vartheta_{\lambda_1}^{m+1} < \vartheta_{\lambda_2}^{m}$, kde



$\sin\vartheta_1 = \frac{(m+1)\lambda_1}{d}$ $\quad \sin\vartheta_2 = \frac{m\lambda_2}{d}$

⇒ teda m-té maximum λ_1 nepresiahne m+1 maximum λ_2

- pre medzný uhol platí (teda $\vartheta_{\lambda_1}^{m+1} = \vartheta_{\lambda_2}^{m}$):

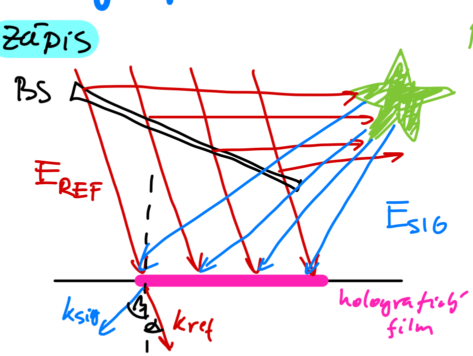
$$h(\sin\vartheta_{FSm} - \sin\vartheta_i) = m\lambda_2 = (m+1)\lambda_1$$

→ voľný interval vlnových dĺžok:

$$F_{Sm} = \lambda_2 - \lambda_1 = \frac{\lambda_1}{m}$$

Holografie

- zápis



$$E_{REF} = A_{REF}\, e^{-i\omega t}\, e^{\overbrace{-ix k_{ref} \sin\alpha}^{\varphi(x)}}$$

$$E_{SIG} = A_{sig}(x)\, e^{-i\omega t}\, e^{\overbrace{-ix k_{sig} \sin\beta(x)}^{\Theta(x)}}$$

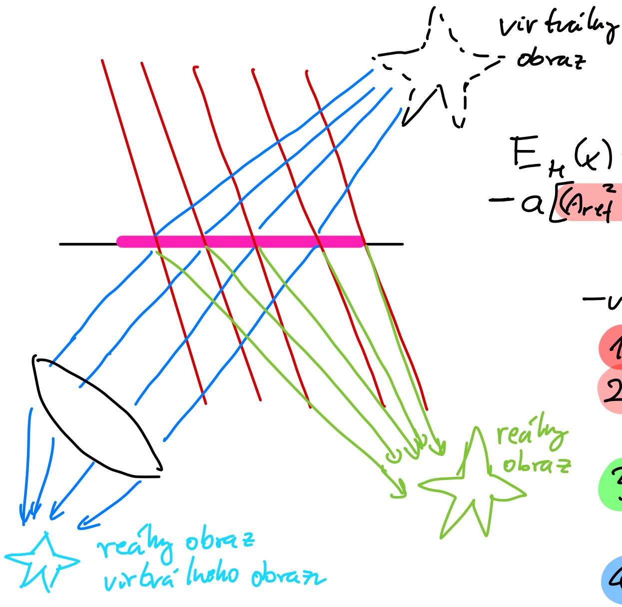
$$I_F(x) = A_{ref}^2 + A_{sig}^2(x) + A_{sig}A_{ref}\underline{e^{-i\varphi}e^{i\Theta}} + A_{sig}A_{ref}\underline{e^{i\varphi}e^{-i\Theta}}$$

→ holografický film zaznamená intezitu el. pole
→ pomocou ref. vlny zaznamenáme aj info o fázy sig. vlny

• čtení

$$t(x) = t_0 - a I_F(x)$$ → amplitudová propustnost

↳ pre neexpon. film



$$E_H(x) = t(x)\, E_{ref} = t_0 E_{ref} -$$
$$- a\left[(A_{ref}^2 + A_{sig}^2) E_{ref} + A_{ref}^2 A_{sig}\, e^{-i(2\varphi-\theta)} e^{-i\omega t} + A_{ref}^2 A_{sig}\, e^{-i\theta} e^{-i\omega t}\right]$$

– vo výsledku budú 4 vlny:

1) prochá zející ref. hologramem
2) ref. vlna, ale amplituda je ovplyvnená hologramom
3) vytvára reálny obraz, ale kde predmet nebol
4) vytvára virtuálny obraz, kde pôvodne bol predmet
↳ pomocou šošovky – reálny predmet

Koherence

korelace záření

Popis koherence

→ sledujeme vlastnosti el. pola vlnového balíka, na rôznych miestach $\vec{r}_1, \vec{r}_2$ a rôznych časoch $t, t+\tau$

→ štatistické vlastnosti el. pola

• kvázimonochromatické žiarenie : $\omega \in (\bar{\omega} - \Delta\omega, \bar{\omega} + \Delta\omega)$ $\Delta\omega \ll \bar{\omega}$

↳ popíšeme fcí: $\vec{E}(\vec{r}, t) = A(t) e^{i(\varphi(t) - \bar{\omega} t)}$

• korelačný funkce – pro stac. prípad:

$$\Gamma_{12}(\vec{r}_1, \vec{r}_2, \tau) = \langle \vec{E}_1(\vec{r}_1, t+\tau)\, \vec{E}_2^*(\vec{r}_2, t) \rangle_{t_D} = \frac{1}{t_D} \int_{-\frac{t_D}{2}}^{\frac{t_D}{2}} \vec{E}_1(\vec{r}_1, t+\tau)\, \vec{E}_2^*(\vec{r}_2, t)\, dt$$

$$I_d(\vec{r}_3) = \frac{1}{4} \varepsilon_0 \varepsilon_r \langle (\vec{E}_1 + \vec{E}_2)(\vec{E}_1^* + \vec{E}_2^*) \rangle_{t_D} =$$

$$= \overbrace{\frac{1}{4} \varepsilon_0 \varepsilon_r \langle \underbrace{|E_1|^2}_{\Gamma_{11}(0)} \rangle_{t_D}}^{I_1} + \overbrace{\frac{1}{4} \varepsilon_0 \varepsilon_r \langle \underbrace{|E_2|^2}_{\Gamma_{22}(0)} \rangle_{t_D}}^{I_2} + \frac{1}{4} \varepsilon_0 \varepsilon_r \underbrace{\langle \vec{E}_1 \vec{E}_2^* + \vec{E}_2 \vec{E}_1^* \rangle_{t_D}}_{2\,\mathrm{Re}\{\Gamma_{12}(\tau)\}}$$

$$I_d(\vec{r}_3) = I_1 + I_2 + \frac{1}{4} \varepsilon_0 \varepsilon_r\, 2\, \mathrm{Re}\{\Gamma_{12}(\tau)\} =$$

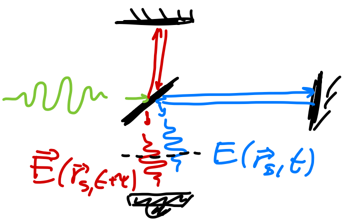
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\, \mathrm{Re}\{\gamma_{12}(\tau)\}$$

$$\gamma_{12}(\vec{r}_1, \vec{r}_2, \tau) = \frac{\Gamma_{12}(\vec{r}_1, \vec{r}_2, \tau)}{\sqrt{\Gamma_{11}(0)\, \Gamma_{22}(0)}}$$ → komplexní stupeň koherence

Časova koherence

• jedno miesto, ale rôzne časy: $\vec{r}_1 = \vec{r}_2 = \vec{r}_s$, $t, t+\tau$

• použijeme Michel. interfer.:

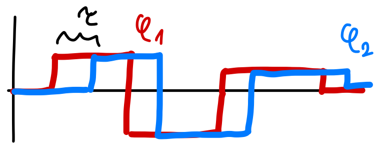


↳ kde $\tau = \frac{d_2 - d_1}{c}$ je časový posun

$$\gamma(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma(0)\, \Gamma(0)}}$$

• skokový model

→ máme jednu vlnu, kt. sa skokovo mení fáza a pošleme do IF aby sme dostali dve posunuté o $\tau = \frac{d_2-d_1}{c}$



$$\Gamma(\tau) = \langle E(\vec r, t+\tau)\, E^*(\vec r, t)\rangle_{t_0} = \langle E_0^2 e^{-i\omega_0\tau} e^{i(\varphi_1-\varphi_2)}\rangle =$$

$$= \frac{E_0^2 e^{-i\omega_0\tau}}{N t_0}\left(\int_0^{t_0-\tau} e^{i0}\,dt + \int_{t_0-\tau}^{t_0} e^{iH_1} + \int_{t_0}^{2t_0-\tau} e^{i0} + \dots\right) =$$

$$= \frac{E_0^2 e^{-i\omega_0\tau}}{N t_0}\left(N(t_0-\tau) + \tau \underbrace{\sum_{n=1}^{N} e^{iH_n}}\right)$$

H_n náhodné $\Rightarrow e^{iH_n} = 0$

$$\Gamma(\tau) = E_0^2 e^{-i\omega_0\tau}\left(1 - \frac{\tau}{t_0}\right)$$

$$\gamma(\tau) = e^{-i\omega_0\tau}\left(1-\frac{\tau}{t_0}\right)$$

$$I = 2I_0\left(1 + \cos(\omega_0\tau)\left(1-\frac{\tau}{t_0}\right)\right)$$

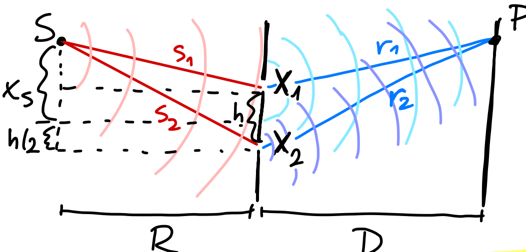
$$I_{max} = 2I_0\left(2-\frac{\tau}{t_0}\right) \qquad I_{min} = 2I_0\left(\frac{\tau}{t_0}\right)$$

$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \left(1-\frac{\tau}{t_0}\right) = |\gamma(\tau)|$$

Prostorová koherence

• popisuje korelaci v jednom čase t, ale na rôznych miestach $\vec r_1, \vec r_2$

• Youngov pokus



- sledujeme porovnáme vlny v bodoch X_1, X_2 interferenciou na Youngovy

↳ dané časovými rozdielmi

$$\tau_s = \frac{s_2-s_1}{c} \qquad \tau_D = \frac{d_2-d_1}{c}$$

$$s_1 = \sqrt{D^2 + \left(x_s - \frac{h}{2}\right)^2} \approx D\left(1 + \frac{x_s^2 - hx_s + h^2}{2D}\right)$$

$$s_2 = \sqrt{D^2 + \left(x_s + \frac{h}{2}\right)^2} \approx D\left(1 + \frac{x_s^2 + hx_s + h^2}{2D}\right)$$

$$s_2 - s_1 = \frac{hx_s}{D}$$

↳ analogicky $d_2 - d_1 = \frac{hx_P}{R}$

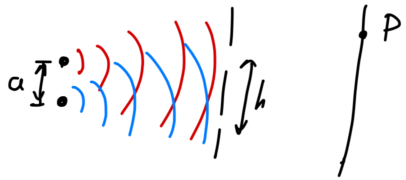
- jeden bod

$$I \propto E_0^2\left(1+\cos(k(d_2-d_1)\right) = E_0^2\left(1+\cos(\omega \tau_D)\right)$$

- dva body

vzdialenost medzi bodmi

$$V = \left|\cos\left(\frac{kah}{2R}\right)\right|$$

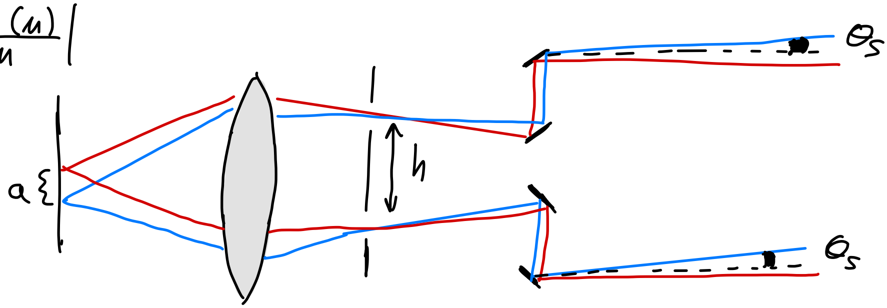


- všechna bodových zdrojov

$$V = \frac{\sin\left(\frac{kah}{2R}\right)}{\frac{kah}{2R}} = \frac{\sin u}{u}$$

- kruhový zdroj – Michel. stelárny IF

$$V = \left|\frac{2J_1(u)}{u}\right|$$



Geometrická optika

Eikonál

- predpokladáme veľkosti objektov $\gg \lambda$

  $\vec{k}\cdot\vec{r} = k_0 n \, \vec{s}_0\cdot\vec{r} = k_0 n$, kde $d = \vec{s}_0\cdot\vec{r}$ je optická dráha

- popis v nehomogénnom prostredí $\Rightarrow$ $\varepsilon = \varepsilon(\vec{r})$, $n = n(\vec{r})$

  $\hookrightarrow$ obecná el. vlna nebude čisto rovinná:

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r})\, e^{ik_0\varphi(\vec{r})}\, e^{-i\omega t}$$
$$\vec{B}(\vec{r},t) = \vec{B}_0(\vec{r})\, e^{ik_0\varphi(\vec{r})}\, e^{-i\omega t}$$

$\varphi(\vec{r})$ je eikonál

Eikonálová rovnice

$\nabla\cdot\vec{D} = 0 \qquad \vec{D} = \varepsilon(\vec{r})\vec{E} \quad \rightarrow$

$$\nabla\cdot\vec{D} = \nabla\cdot(\varepsilon(\vec{r})\vec{E}) = \varepsilon(\vec{r})\nabla\cdot\vec{E}_0 + ik_0\varepsilon(\vec{r})\vec{E}_0\cdot\nabla\varphi(\vec{r}) + \vec{E}_0\cdot\nabla\varepsilon(\vec{r}) = 0$$

$$\nabla\cdot\vec{B} = 0 \quad \rightarrow \quad \nabla\cdot\vec{B}_0(\vec{r}) + ik_0\vec{B}_0\cdot\nabla\varphi(\vec{r}) = 0$$

$$\nabla\times\vec{E} = \varepsilon(\vec{r})\nabla\times\vec{E}_0 + ik_0\varepsilon(\vec{r})\nabla\varphi(\vec{r})\times\vec{E}_0 = -\frac{\partial\vec{B}}{\partial t} = i\omega\vec{B}_0$$

$$\nabla\times\vec{H} = \frac{1}{\mu_0}\nabla\times\vec{B}_0 + ik_0\frac{1}{\mu_0}\nabla\varphi(\vec{r})\times\vec{B}_0 = -i\omega\,\varepsilon(\vec{r})\vec{E}_0$$

- predpokladáme $\lambda_0 \rightarrow 0$, teda $k_0 = \frac{2\pi}{\lambda_0} \rightarrow \infty$ a dominujú členy

$$\vec{E}_0\cdot\nabla\varphi(\vec{r}) = 0$$
$$\vec{B}_0\cdot\nabla\varphi(\vec{r}) = 0$$

$$\vec{B}_0 = \frac{k_0}{\omega}\nabla\varphi(\vec{r})\times\vec{E}_0$$
$$\vec{E}_0 = \frac{-k_0}{\omega\varepsilon}\nabla\varphi(\vec{r})\times\vec{B}_0$$

$$\vec{E}_0 = -\underbrace{\frac{k_0^2}{\omega^2\varepsilon\mu_0}}_{\frac{1}{n^2}}\nabla\varphi\times\nabla\varphi\times\vec{E}_0 = -\frac{1}{n^2}\left(\nabla\varphi(\cancel{\nabla\varphi\cdot\vec{E}_0}) - \vec{E}_0(\nabla\varphi\cdot\nabla\varphi)\right)$$

$$n^2\vec{E}_0 = \vec{E}_0(\nabla\varphi)^2 \Rightarrow (\nabla\varphi)^2 = n^2 \Rightarrow \nabla\varphi = n\,\vec{s}_0$$

eikonálová rovnica

Paprsková rovnice

- paprsek = parametrická krivka, ktorej je $\vec{s}_0(\vec{r})$ tečna
  ↳ jej param. vyjadrenie $\vec{r}(s)$, kde s je param
  → potom platí: $$\frac{d\vec{r}(s)}{ds} = \vec{s}_0(\vec{r})$$
  → teda z eikonál. rov: $\nabla \varphi = n(\vec{r})\vec{s}_0 = n(\vec{r})\frac{d\vec{r}}{ds}$
  → zderivujeme:

$$\frac{d}{ds}\left[n(\vec{r}(s))\,\vec{s}_0(\vec{r}(s))\right] = \nabla n(\vec{r})$$

Lagrangeov invariant

$$\nabla \varphi = n(\vec{r})\cdot\vec{s}_0(\vec{r}) \rightarrow \nabla\times\nabla\varphi = 0 = \nabla\times\left(\vec{n}(\vec{r})\cdot\vec{s}_0(\vec{r})\right)$$

$$\Rightarrow 0 = \int \nabla\times\left(n(\vec{r})\cdot\vec{s}_0(\vec{r})\right)\cdot d\vec{S} = \oint_l n(\vec{r})\,\vec{s}_0(\vec{r})\cdot d\vec{l} = 0$$

$$\oint_l n(\vec{r})\,\vec{s}_0(\vec{r})\cdot d\vec{l} = 0$$ → Lag. invariant

Fermatov princip

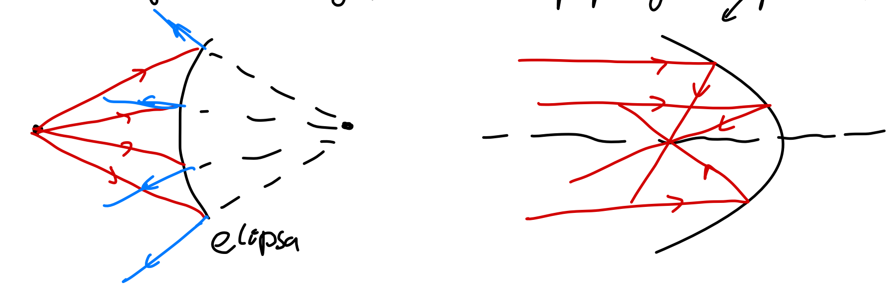
Svetlo sa medzi dvoma bodmi šíri po najkratšej optickej dráhe.

$$T = \int_A^B dt = \int_A^B \frac{ds}{v(\vec{r})} = \int_A^B \frac{n(\vec{r})}{c}\,ds$$

↳ čas musí byť najkratší: $\delta T = \delta\int_A^B n(\vec{r})\,ds = 0$

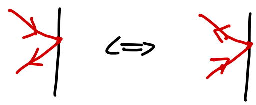
Descartovy plochy

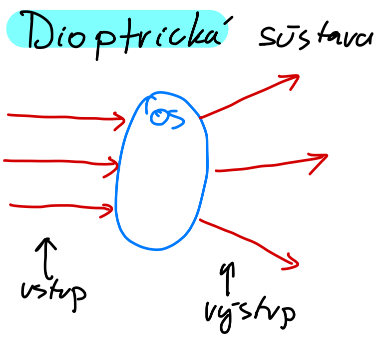
- sformulujeme rovinu aby fokusovali paprsky

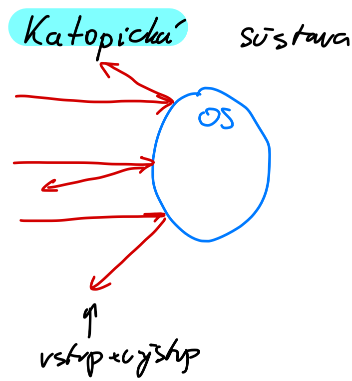


Paraxiálna optika

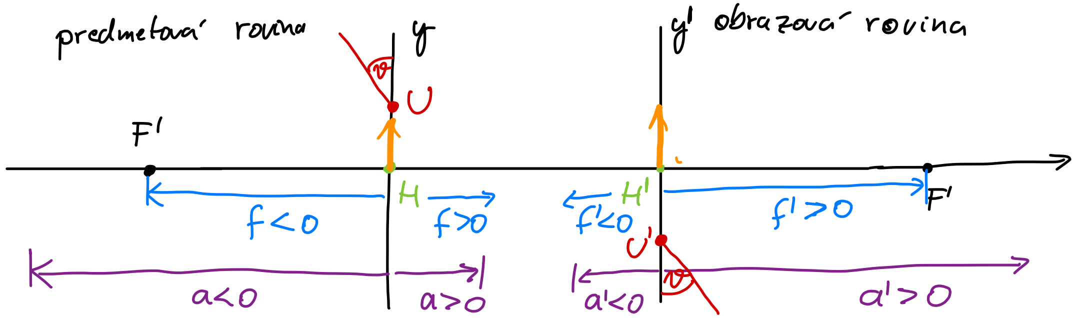
- predpokladame:
  1) platnosť paprskovej rovnice
  2) zákon odrazu a lomu
  3) paprsek nezávisí na iných paprskoch
  4) paprsek nezavisí na volbe šípky:
  5) kolineárne transformácie opt. prístrojov







Gaussova zobrazovacia rovnica



Kardinálne body:

H, H' .... hlavné body

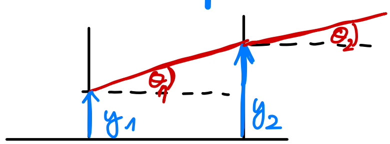
F, F' ... ohniskové body

U, U' ... uzlové body (uhlové zväčšenie konjugovaných priamok je 1)

$$\frac{f}{a}+\frac{f'}{a'}=1$$

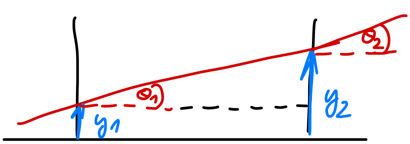
$$\frac{x'}{x}=\frac{y'}{y}=\frac{f}{f-a}=\frac{f'-a'}{f'}$$

Maticová optika



$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{\text{prenosová matica daná OS}} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

- šírenie medzi rovinami

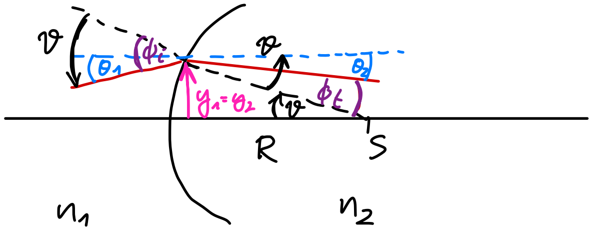


$$\theta_1 = \theta_2$$

$$y_2 = y_1 - \operatorname{tg}\theta_1\, d \approx y_1 - \theta_1 d$$

$$\rightarrow \begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & -d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

- lom na kulovej ploche



$$n_1 \phi_i = n_2 \phi_t$$

$$\sin\vartheta = \frac{y_1}{R} \approx \vartheta$$

$$\vartheta = \theta_1 + \phi_i \qquad \frac{y_1}{R} = \theta_1 + \phi_i$$

$$\vartheta = \theta_2 + \phi_t \qquad \frac{y_1}{R} = \theta_2 + \frac{n_1}{n_2}\phi_i \quad \Rightarrow \frac{y_1}{R} = \theta_2 + \frac{n_1}{n_2}\left(\frac{y_1}{R} - \theta_1\right)$$

$$\Rightarrow \theta_2 = \left(1 - \frac{n_1}{n_2}\right)\frac{y_1}{R} + \frac{n_1}{n_2}\theta_1 \quad \Rightarrow \begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \left(1-\frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

↳ špec. prípad: $\theta_1 = 0$, $\theta_2 = \frac{y}{f'}$

$$\Rightarrow \frac{1}{f'} = \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} \quad \Rightarrow T = \begin{pmatrix} 1 & 0 \\ \frac{1}{f'} & \frac{n_1}{n_2} \end{pmatrix}$$

- gaussova zob. rovnica

$$T = \underbrace{\begin{pmatrix} 1 & -a' \\ 0 & 1 \end{pmatrix}}_{\text{prenos } 1} \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{1}{f'} & \frac{n_1}{n_2} \end{pmatrix}}_{\text{lom}} \underbrace{\begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix}}_{\text{prenos } 2} = \begin{pmatrix} 1 & -a' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -a \\ \frac{1}{f'} & -\frac{a}{f'} + \frac{n_1}{n_2} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 - \frac{1}{f'} & -a + \frac{aa'}{f'} - a'\frac{n_1}{n_2} \\ \frac{1}{f'} & -\frac{a}{f} + \frac{n_1}{n_2} \end{pmatrix}$$

→ podmienka je aby sa papršky sťahali ⇒ y_2 nesmie závisieť na θ_1

$$\Rightarrow -a + \frac{aa'}{f'} - a'\frac{n_1}{n_2} = 0 \;\rightarrow\; a = a'\left(\frac{a}{f'} - \frac{n_1}{n_2}\right) = a'\left(\frac{a}{f'} - \frac{f}{f'}\right)$$

$$\Rightarrow af' = a'(a - f) \Rightarrow \frac{a-f}{a} = \frac{f'}{a'} \rightarrow 1 - \frac{f}{a} = \frac{f'}{a'} \Rightarrow 1 = \frac{f}{a} + \frac{f'}{a'}$$

• tenké čočky

$$T = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_2'} & \frac{n_2}{n_3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f_1'} & \frac{n_1}{n_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_2'} + \frac{1}{f_1'}\frac{n_2}{n_3} & \frac{n_1}{n_3} \end{pmatrix}$$

$$\frac{1}{f_1'} = \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R_1}$$

$$\frac{1}{f'} = \frac{1}{f_2'} + \frac{1}{f_1'}\frac{n_2}{n_3}$$

$$\frac{1}{f_2'} = \left(1 - \frac{n_2}{n_3}\right)\frac{1}{R_2}$$

pro $n_1 = n_3 = 1$, $n_2 = n$:

$$\frac{1}{f'} = (1-n)\frac{1}{R_2} + (n-1)\frac{1}{R_1} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)(n-1) = -\frac{1}{f}$$

• odraz na zrcadle

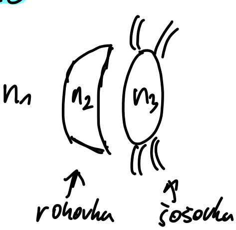
$$\frac{f}{a} + \frac{f'}{a'} = 1 \Rightarrow f = f' = \frac{R}{2} \Rightarrow \frac{1}{a} + \frac{1}{a'} = \frac{2}{R}$$

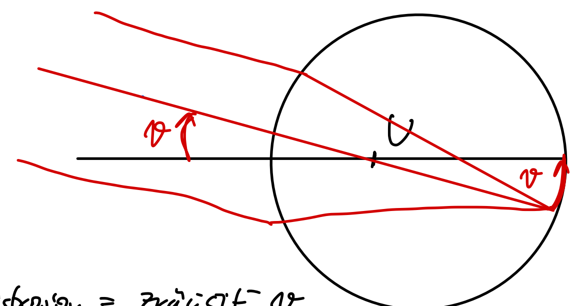
Zväčšenie

- priečne $M_T = \frac{y'}{y} = A = \frac{f'-a'}{f'} = \frac{f}{f-a}$
- pozdĺžne $M_L = \frac{\Delta a'}{\Delta a}$
- uhlové $M_\theta = \frac{\mathrm{tg}\,\theta'}{\mathrm{tg}\,\theta} \approx \frac{\theta'}{\theta} = D$

Optické prístroje

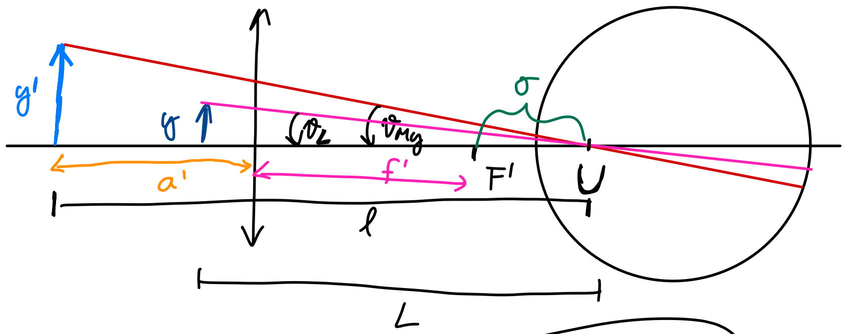
Oko





→ pomocou opt. prístrojov = zväčšiť ϑ

Lupa



$$f' - a' = -(l + \sigma)$$

$$\frac{y'}{y} = \frac{f' - a'}{f'} = -\frac{l + \sigma}{f'}$$

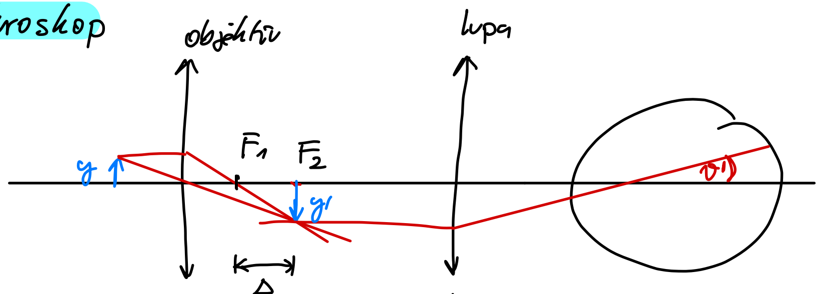
$$\vartheta_L = -\frac{y}{L}$$

$$\vartheta_{MG} = -\frac{y'}{l}$$

$$\Gamma_{MG} = \frac{\vartheta_{MG}}{\vartheta_L} = \frac{-\frac{y'}{L}}{-\frac{y}{l}} = \frac{y'}{y}\frac{L}{l} = -\frac{l+\sigma}{f'}\frac{L}{l} = \frac{L}{f}\left(1 + \frac{\sigma}{l}\right)$$

Γ zvětšení

Mikroskop

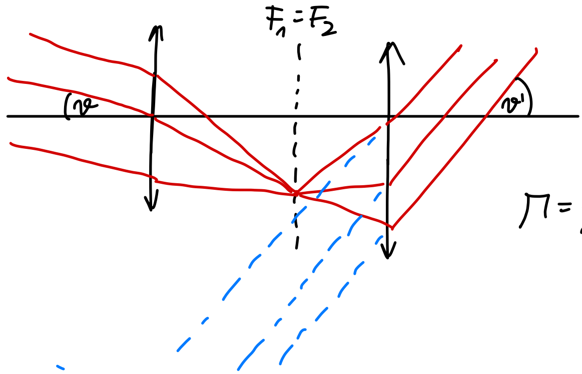


$$\vartheta_L = -\frac{y}{L} \qquad \vartheta' = \frac{y'}{f_2} \qquad \Gamma = \frac{\frac{y'}{f_2}}{-\frac{y}{L}} = -\frac{y'}{y}\frac{L}{f_2} = -\frac{\Delta}{f_1'}\frac{L}{f_2'}$$

$$\frac{y'}{y} = -\frac{\Delta}{f_1'}$$

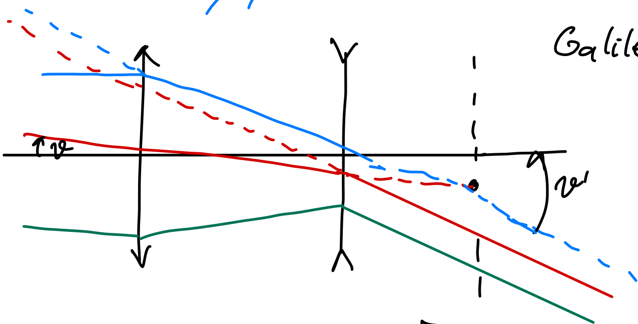
Dalekohlad

Kepler

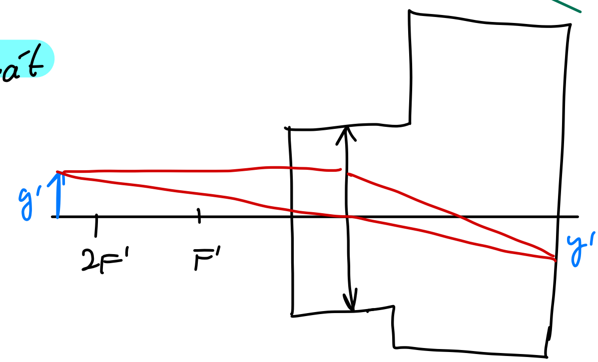


$$\Gamma = \frac{\vartheta'}{\vartheta} = \frac{\frac{y}{f_1'}}{\frac{y'}{f_2}} = \frac{f_1'}{f_2}$$

Galileo



Fotoaparát



Anizotropné prostredie

Základný popis

$$\vec{P} = \varepsilon_0 \overleftrightarrow{\chi} \vec{E} = \varepsilon_0 (1 - \overleftrightarrow{\varepsilon}_r) \vec{E}$$

$$\varepsilon_r = \begin{pmatrix} n_1^2 & & \\ & n_2^2 & \\ & & n_3^2 \end{pmatrix} \rightarrow \text{predpokladáme diagonalizovanú maticu}$$

$$\left.\begin{aligned} D_x &= \varepsilon_0 n_1^2 E_x = \varepsilon_0 n_1^2 E_{0x} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ D_y &= \varepsilon_0 n_2^2 E_y = \varepsilon_0 n_2^2 E_{0y} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ D_z &= \varepsilon_0 n_3^2 E_z = \varepsilon_0 n_3^2 E_{0z} e^{i(\vec{k}\cdot\vec{r}-\omega t)} \end{aligned}\right\} \text{vidíme, že obecne } \vec{D} \nparallel \vec{E}$$

$$\nabla \cdot \vec{D} = i \vec{k} \cdot \vec{D} = 0 \Rightarrow \vec{k} \perp \vec{D}$$

$$\nabla \cdot \vec{D} = \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = i \vec{k} (\varepsilon_0 \vec{E} + \vec{P}) = 0$$

$$\Rightarrow \text{obecne } \vec{k} \cdot \vec{E} \neq 0 \Rightarrow \vec{k} \text{ nemí kolmé na } \vec{E}$$

Fresnelova rovnice

$$\vec{k} \times \vec{H} = -\omega \vec{D} = -\omega \varepsilon_0 \overleftrightarrow{\varepsilon}_r \vec{E}$$

$$\vec{k} \times \vec{E} = \omega \vec{B} = \omega \mu_0 \vec{H}$$

$$\vec{k} \times \vec{k} \times \vec{E} = -\omega^2 \overbrace{\varepsilon_0 \mu_0}^{c} \overleftrightarrow{\varepsilon}_r \vec{E}$$

$$\vec{k} (\vec{k} \cdot \vec{E}) - k^2 \vec{E} = -\omega^2 c \overleftrightarrow{\varepsilon}_r \vec{E}$$

$$k^2 \vec{s}_0 (\vec{s}_0 \cdot \vec{E}) - k^2 = -\omega^2 c \overleftrightarrow{\varepsilon}_r \vec{E} \Rightarrow n^2 \vec{s}_0 (\vec{s}_0 \cdot \vec{E}) - n^2 \vec{E} = -\overleftrightarrow{\varepsilon}_r \vec{E}$$

$$\Rightarrow \begin{aligned} (n^2 - n_1^2) E_x &= n^2 s_{0x} (\vec{s}_0 \cdot \vec{E}) \\ (n^2 - n_2^2) E_y &= n^2 s_{0y} (\vec{s}_0 \cdot \vec{E}) \\ (n^2 - n_3^2) E_z &= n^2 s_{0z} (\vec{s}_0 \cdot \vec{E}) \end{aligned} \qquad (*)$$

$$\Rightarrow \frac{S_{0x}^2}{n^2-n_1^2} + \frac{S_{0y}^2}{n^2-n_2^2} + \frac{S_{0z}^2}{n^2-n_3^2} = \frac{1}{n^2}$$ → Fresnelova rovnica

↳ alternativny zápis:

$$(n^2-n_2^2)(n^2-n_3^2)n^2S_{0x}^2 + (n^2-n_1^2)(n^2-n_3^2)n^2S_{0y}^2 + (n^2-n_1^2)(n^2-n_2^2)n^2S_{0z}^2 = (n^2-n_1^2)(n^2-n_2^2)(n^2-n_3^2)$$

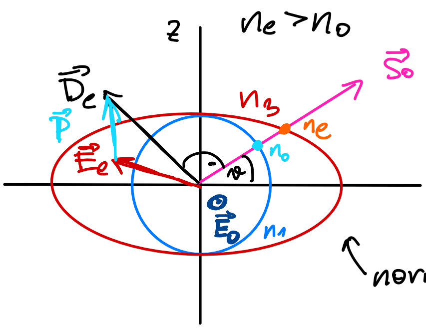
- n je neznáma, určuje rýchlosť svetla
  ↳ obecne môže mať viacej riešení
- pre $n_1 = n_2$ → jednoosé materiály, osa z je optická osa
  ↳ z rovnice dostaneme:

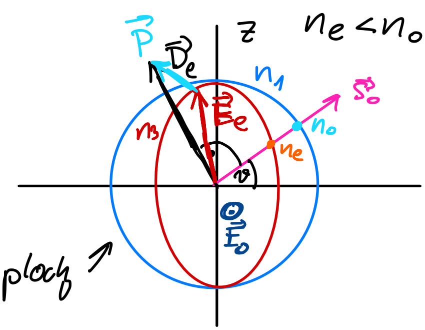
$$(n-n_1^2)\left[-(n^2-n_1^2)(n^2-n_3^2) + n^2\left((n^2-n_3^2)S_{0x}^2 + (n^2-n_3^2)S_{0y}^2 + (n-n_1^2)S_{0z}^2\right)\right] = 0$$

→ prvé riešenie je $n_o^2 = n_1^2$ → tzv. ordinárna vlna
→ nezávisí na $\vec{S}_0$

→ druhé riešenie je extraordinárna vlna n_e, v reze $y=0$ dostaneme:

$$\frac{1}{n_e^2} = \frac{\sin^2\vartheta}{n_3^2} + \frac{\cos^2\vartheta}{n_1^2} \rightarrow 1 = \frac{x^2}{n_3^2} + \frac{z^2}{n_1^2} \xrightarrow{3D} 1 = \frac{x^2}{n_3^2} + \frac{y^2}{n_3^2} + \frac{z^2}{n_1^2}$$

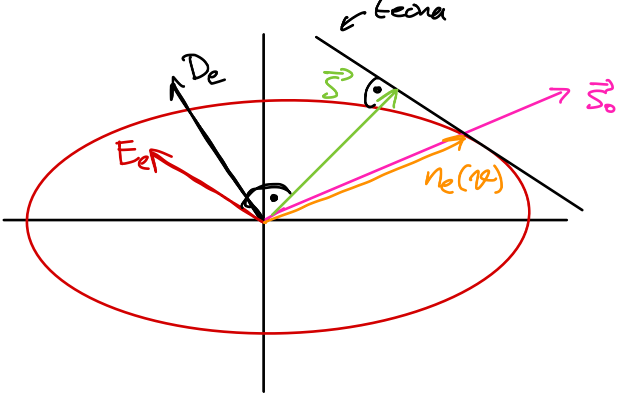




normálové plochy

1) pre $n = n_1 = n_o$ z (*) dostaneme, že $E_z = 0, E_x = 0$, lin. pol.
2) pre $n = n_3 = n_e$ – vlna je lin. polarizovaná

Poyntingov vektor

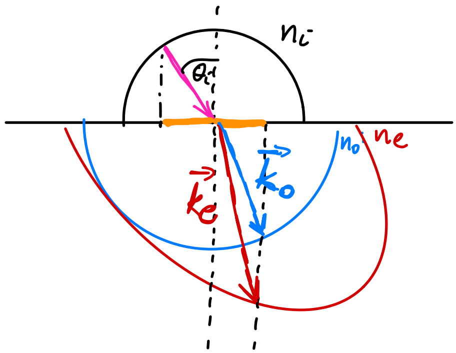


- $\vec{S}$ je tečné na normálu plochy
- $\vec{S} \perp \vec{E}_e$ (z def.)

normála

$$\vec{N} = \left(\frac{2 n_e \sin\vartheta}{n_3^2},\ 0,\ \frac{2 n_e \cos\vartheta}{n_1^2}\right)$$

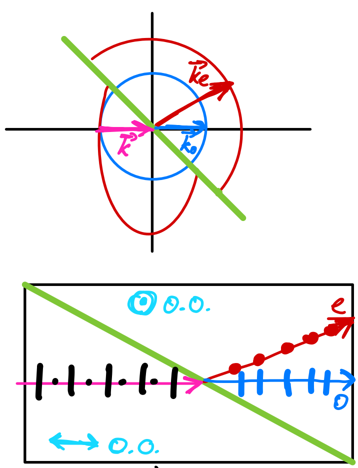
Lom svetla



- riadna vlna: $n_1 \sin\theta_i = n_o \sin\theta_{to}$
- mimoriadna vlna: $n_1 \sin\theta_i = n_e(\theta_{te}) \sin\theta_{te}$

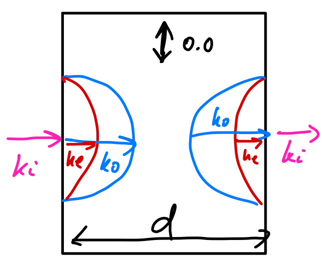
Použitie

Roscheov polhranol



→ rozdelenie vlny na 2 polariz.

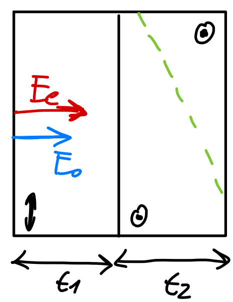
Fázová destička



- posun fáze:

$$\Delta\varphi = \frac{2\pi}{\lambda} d(n_e - n_o)$$

Kompenzátor



- obecný nastaviteľný posun fáze

$$\Delta\varphi = \frac{2\pi}{\lambda}(n_e - n_o)(t_2 - t_1)$$

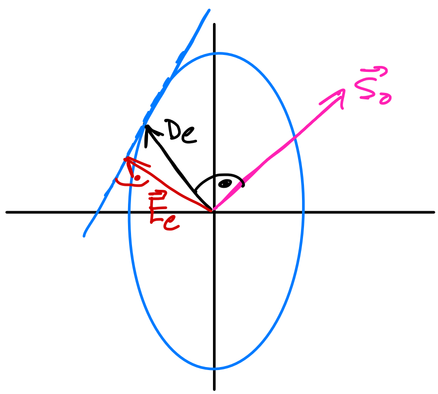
Indikatrix

– alternatívny popis, kedy $n = n(\vec{D})$ a nie $n = n(\vec{s}_0)$

$$w_e = \frac{1}{2} E \cdot D = \frac{1}{2}\left(E_x D_x + E_y D_y + E_z D_z\right) = \frac{1}{2}\left(\frac{D_x^2}{\varepsilon_1} + \frac{D_y^2}{\varepsilon_2} + \frac{D_z^2}{\varepsilon_3}\right)$$

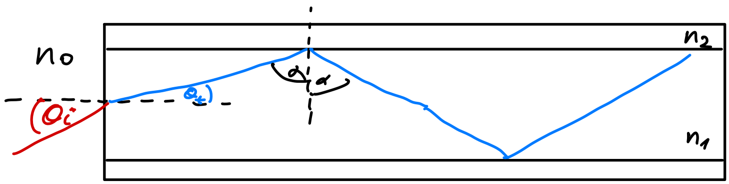
$$\Rightarrow \frac{D_x^2}{n_1^2} + \frac{D_y^2}{n_2^2} + \frac{D_z^2}{n_3^2} = 1$$, kde sme normalizovali $2 w_e \varepsilon_0 = 1$

→ jeden elipsoid dá info o ord. aj extraord. vlne



Vláknová optika

Vedenie vlny



→ totálny odraz

- medzný uhol: $n_1 \sin\alpha_c = n_2 \rightarrow \sin\alpha_c = \frac{n_2}{n_1}$

$$n_0 \sin\theta_i = n_1 \sin\theta_t = n_1 \cos\alpha_c$$

$\theta_t + \alpha_c = \frac{\pi}{2}$

$$\Rightarrow \left.\begin{aligned} \left(\frac{n_0}{n_1}\right)^2 \sin^2\theta_i &= \cos^2\alpha_c \\ \left(\frac{n_2}{n_1}\right)^2 &= \sin^2\alpha_c \end{aligned}\right] + \rightarrow 1 = \frac{n_0^2}{n_1^2}\sin^2\theta_i + \frac{n_2^2}{n_1^2}$$

$$\Rightarrow \left(1 - \frac{n_2^2}{n_1^2}\right)\frac{n_1^2}{n_0^2} = \sin^2\theta_i \Rightarrow n_0^2 \sin^2\theta_i = n_1^2 - n_2^2$$

$$\Rightarrow \sin\theta_i = \frac{NA}{n_0} \qquad NA = \sqrt{n_1^2 - n_2^2}$$

Módy

$$\Delta\varphi = k\,2d\cos\alpha + \Delta\varphi_{odr} \approx 2n_1 k d \cos\alpha = 2m\pi$$

$$\left[\frac{2d}{\lambda_0} n_1 \cos\alpha_c\right] = \left[\frac{2d}{\lambda_0} NA\right] = M$$

- $M+1$ udáva počet módov, kt. sa môže šíriť vlna

Útlm

$$B = 10\log_{10}\frac{P_1}{P_2} \rightarrow \text{útlm}$$

P_1 → výkon na vstupe

P_2 → výkon na výstupe

Nelineárna optika

- rozvoj vzťahu $P_i = \varepsilon_0 \chi_{ij}(\omega_j) E_j(\omega_j)$

$$P_i = \varepsilon_0 \chi_{ij}(\omega_j) E_j(\omega_j) + \underbrace{\varepsilon_0 \chi_{ijk}(\omega_i, \omega_j, \omega_k) E_j(\omega_j) E_k(\omega_k)}_{\text{2. rádu}} + \underbrace{\varepsilon_0 \chi_{ijkl}(\omega_i, \omega_j, \omega_k, \omega_l) E_j(\omega_j) E_k(\omega_k) E_l(\omega_l)}_{\text{3 rádu}}$$

2. rádu

$$P_i^{(2)} = \varepsilon_0 \chi_{ijk} E_j E_k$$

- pre $E = E_0\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right) = E_0 \cos\omega t$ a izotropné prostredie:

$$P^{(2)} = \varepsilon_0 \frac{E_0^2}{4}\left(e^{2i\omega t} + e^{-2i\omega t} + 1\right) = \underbrace{\varepsilon_0 \frac{E_0^2}{4}}_{\text{optické usmernenie}} + \underbrace{\varepsilon_0 \frac{E_0^2}{2}\cos(\omega t)}_{\text{vlna 2. rádu}}$$

→ generovanie druhej harmonickej

→ konštantná polarizácia

- podmienka stáčovania: $2k_1 = k_2 \rightarrow 2\omega \frac{n(\omega)}{c} = 2\omega \frac{n(2\omega)}{c}$

$$\Rightarrow n(\omega) = n(2\omega)$$

3. rádu

$$P^{(3)} = \varepsilon_0 \frac{E_0^3}{8}\left(e^{-i\omega t} + e^{i\omega t}\right)^3 = \varepsilon_0 \frac{E_0^3}{8}\left(e^{-i3\omega t} + e^{i3\omega t} + 3e^{-i\omega t} + 3e^{i\omega t}\right)$$

$$= \underbrace{\varepsilon_0 \frac{E_0^3}{4}\cos(3\omega t)}_{P^{(3)}_{3\omega}} + \underbrace{\frac{3}{4}\varepsilon_0 E_0^3 \cos(\omega t)}_{P^{(3)}_{\omega}}$$

třetí harmonická

intenzita

$$P^{(3)}_{\omega} = \varepsilon_0 \frac{3}{4} E_0^2 \left(E_0 \cos(\omega t)\right) \propto I \, E(t)$$

$$P_{\omega} = P^{(1)}_{\omega} + P^{(3)}_{\omega} = \varepsilon_0 \chi_L E(t) + \varepsilon_0 \chi_{NL} E(t)$$

$$\Rightarrow n = n_L + n_{NL} = n_L + \frac{1}{2} n_2 I \qquad ; n_2 = \frac{3\chi^{(3)}}{2 n_L^2 \varepsilon_0 c}$$

$\Rightarrow$ samofokusace a automodulace

Interakce záření s hmotou

Základní popis

$$I = I_0 e^{-\alpha z}$$, α – extinční koef.

↳ při absorpci = absorpční koeficient

$$E_x = E_0 e^{-k_I z} e^{i(k_R z - \omega t)} = E_0 e^{i(\tilde{k} z - \omega t)}$$

$\tilde{k} = k_R + i k_I$ → cplx. vlnový vektor

$$\left.\begin{aligned} k_R &= \frac{\omega}{c} n \\ k_I &= \frac{\omega}{c} \kappa \end{aligned}\right\} \tilde{N} = n + i\kappa$$ → cplx. index lomu

$$\nabla \times E = -\frac{\partial}{\partial t}\vec{B} \rightarrow \frac{\partial E_x}{\partial z} = -\frac{\partial}{\partial t} B_y$$

$$\rightarrow B_y = E_0 \frac{\tilde{k}}{\omega} e^{i(\tilde{k}z - \omega t)} = \frac{E_0}{\omega} |\tilde{k}| e^{i\varphi_N} e^{i(\tilde{k}z - \omega t)} = B_0 \underbrace{e^{i\varphi_N}}_{} e^{i(\tilde{k}z - \omega t)}$$

fázový posun mezi $\vec{B}$ a $\vec{E}$

$\Rightarrow \vec{B}$ a $\vec{E}$ nekmitají ve fázi $\qquad \operatorname{tg} \varphi_N = \frac{\kappa}{n}$

$$H_y = \varepsilon_0 c |\tilde{N}| e^{i\varphi_N} E_x \qquad \varepsilon_r = \tilde{N}^2$$

$$D_x = \varepsilon_0 |\tilde{N}|^2 e^{i2\varphi_N} E_x$$

↳ posun o $2\varphi_N$

$$P_x = \varepsilon_0 |\tilde{\chi}| e^{i\varphi_\chi} E_x$$

$$\tilde{\chi} = \chi_R + i\chi_I$$

$$\varepsilon_R = \chi_R + 1$$

$$\varepsilon_I = \chi_I$$

Fresnelovy koeficienty

$$\theta_i = \theta_t = 0 \quad \Rightarrow \quad r_{s,p} = r = \frac{n_1 - n_2}{n_1 + n_2}$$

pro $n_1 = 1 \Rightarrow \underbrace{r = \frac{1-n_2}{1+n_2}}_{\text{neabsorbující}} \quad \Rightarrow \underbrace{r = \frac{1-n_2-i\kappa_2}{1+n_2+i\kappa_2}}_{\text{absorbující}}$

$$R = |r|^2 = \frac{1-n_2-i\kappa_2}{1+n_2+i\kappa_2} \, \frac{1-n_2+i\kappa_2}{1+n_2-i\kappa_2} = \boxed{\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2} = R}$$

Poyntingův vektor

$$\vec{S} = \vec{E} \times \vec{H} = \mathrm{Re}\{\vec{E}\} \times \mathrm{Re}\{\vec{H}\} = \frac{1}{2}(E_x + E_x^*)\frac{1}{2}(H_y + H_y^*)$$

$$\langle S_z \rangle_T = \frac{1}{4}\langle \cancelto{0}{E_x H_y} + E_x^* H_y + E_x H_y^* + \cancelto{0}{H_y^* E_x^*} \rangle_T =$$

v integrály vymizí
$\int_0^T \cos(2\omega t) = 0$

$$= \frac{1}{4}\langle E_x^* H_y + E_x H_y^* \rangle = \frac{1}{4} E_0^2 \varepsilon_0 c \langle e^{i(\tilde{k}z-\omega t)}(n-i\kappa)e^{-i(\tilde{k}^*z-\omega t)} + e^{-i(\tilde{k}z-\omega t)}(n+i\kappa)e^{i(\tilde{k}^*z-\omega t)} \rangle =$$

$$= \frac{1}{4} E_0^2 \varepsilon_0 c \, e^{-2k_I z}[n - i\kappa + n + i\kappa] = \underbrace{\frac{1}{2} E_0^2 \varepsilon_0 c n}_{I_0} e^{-2k_I z} = I_0 e^{-2k_I z} = I_0 e^{-\alpha z}$$

$$\Rightarrow \boxed{\alpha = 2k_I}$$

Lorentzov model

- model dielektrika
- elektrón = LHO

$$\ddot{x}(t) + \gamma \dot{x}(t) + \omega_0^2 x(t) = \frac{q}{m} E(t) = \frac{q}{m} E_0 e^{-i\omega t}$$

$$\omega_0^2 = \frac{k_H}{m}$$

$x(t) = x_0 e^{-i\omega t}$:

$$-\omega^2 x_0 - i\omega\gamma x_0 + \omega_0^2 x_0 = \frac{q}{m} E_0$$

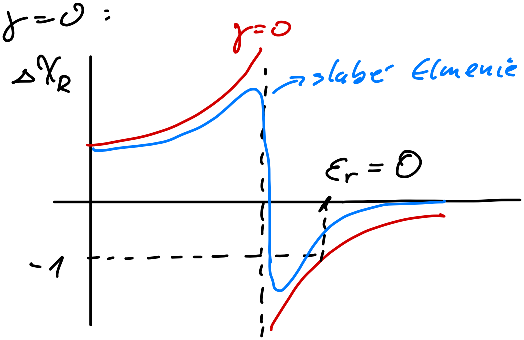
$$\Rightarrow \quad x_0 = \frac{q/m\, E_0}{\omega_0^2 - i\omega\gamma - \omega^2}$$

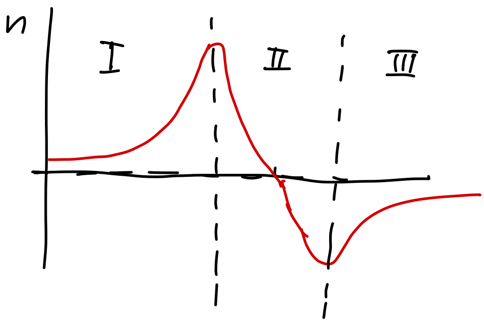
$$P = qNx = \frac{q^2 N}{m} \frac{\cancel{E_0 e^{-i\omega t}}}{\omega_0^2 - i\omega\gamma - \omega^2} = \varepsilon_0 \chi \cancel{E}$$

$$\Delta\chi = \frac{\frac{q^2 N}{\varepsilon_0 m}}{\omega_0^2 - i\omega\gamma - \omega^2} = \underbrace{\frac{Nq^2}{\varepsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2}}_{\Delta\chi_R} + i \underbrace{\frac{Nq^2}{\varepsilon_0 m} \frac{\gamma m}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}_{\Delta\chi_I}$$

- 1 oscilátor bez trenia, $\gamma = 0$:

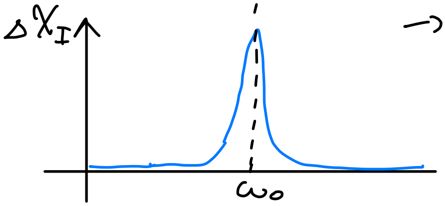
$$\Delta\chi_R = \frac{q^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2}$$





I, III ... oblast normální disperze

II ... oblast anomální disperze



→ tlmenie najväčšie v okolí ω_0

Drudeho model

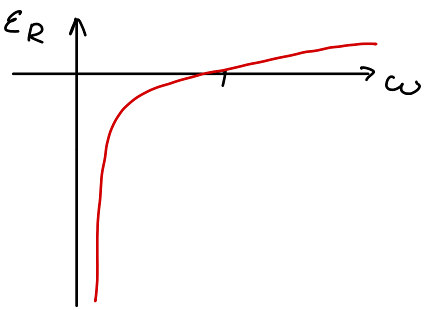
• voľné nosiče $\Rightarrow k_H = 0 \Rightarrow \omega_0 = 0$

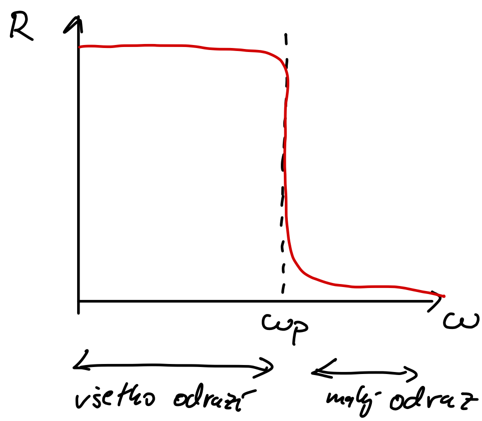
$$\tilde{\varepsilon}_r = 1 + \frac{Nq^2}{\varepsilon_0 m}\,\frac{1}{-i\gamma\omega - \omega^2} = 1 - \frac{\omega_p^2}{i\gamma\omega + \omega^2}$$

$$\omega_p^2 = \frac{Nq}{\varepsilon_0 m}$$

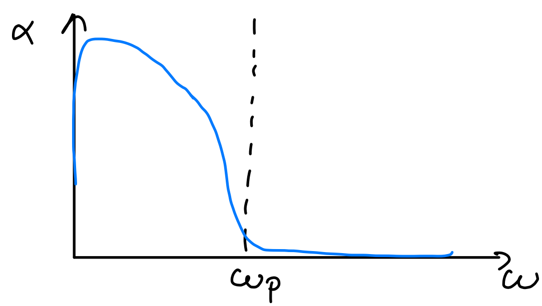
plazmová frek.

$$\tilde{\varepsilon}_r = \underbrace{1 - \frac{\omega_p^2\,\omega^2}{\omega^4 + \gamma^2\omega^2}}_{Re} + i\,\underbrace{\frac{\gamma\,\omega_p^2}{\omega^4 + \gamma^2\omega^2}}_{Im}$$





→ pre $\omega > \omega_p$ kov neodráža žiarenie ⇒ presvitenie kovu



Žáření černého tělesa

Žárení černého telesa

$$I_0(\nu,T) = I_r(\nu,T) + I_a(\nu,T)$$

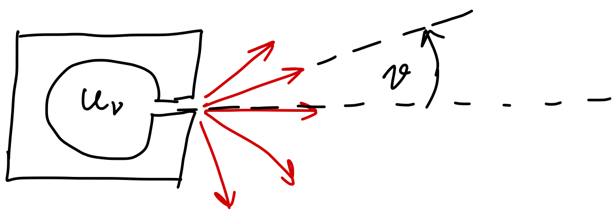
$$1 = \frac{I_r}{I_0} + \frac{I_a}{I_0} = \rho + \alpha$$

$\varepsilon = \frac{I_e}{I_0}$ → emisivita $\qquad \frac{\varepsilon}{\alpha}$ = konst. → nezávisí na materiály

→ černé těleso vše absorbuje, $\alpha_{BB} = 1$

$$\frac{\varepsilon}{\alpha} = \varepsilon_{BB}$$

$u_\nu(\nu,T)$ ... objémová hustota energie žiarení na 1 Hz



$$B_\nu(\nu,T,\vartheta) = \frac{c}{4\pi} u_\nu(\nu,T) \overbrace{f(\vartheta)}^{\cos\vartheta}$$

Wienov zákon

$u_\nu(\nu,T) = A\nu^3 e^{-\frac{B\nu}{T}}$ → platí iba pre veľké ν

⇒ IF katastrofa

Rayleighův - Jeansův zákon

$$u_\nu = \frac{8\pi\nu^2}{c^3} k_B T$$

$\rightarrow$ platí len pre malé ν

$\Rightarrow$ UV katastrofa

Planckov zákon

$$u_\nu = \frac{8\pi\nu^2}{c^3} \frac{b'\nu}{e^{\frac{a'\nu}{T}} - 1}$$

$\rightarrow$ platí pre všetky ν

$\rightarrow$ empirické konšt. a', b'

$\downarrow$ použitím úplne iného modelu

$$u_\nu = \frac{8\pi\nu^2}{c^3} \frac{\varepsilon}{e^{\frac{\varepsilon}{k_B T}} - 1}$$

$\Rightarrow \varepsilon = b'\nu = h\nu$

$\frac{a'\nu}{T} = \frac{\varepsilon}{k_B T} \rightarrow a' = \frac{h}{k_B}$

$\Rightarrow$ úplne správny vzťah:

$$u_\nu(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$\hookrightarrow (R-J) \Rightarrow \frac{8\pi\nu^2}{c^3} \frac{\frac{h\nu}{k_B T}}{e^{\frac{h\nu}{k_B T}} - 1} \cdot k_B T \approx \frac{8\pi\nu^2}{c^3} k_B T$$

$\rightarrow 1$ pro $\nu \rightarrow 0$

$$\hookrightarrow (W) \Rightarrow \frac{8\pi h}{c^3} \nu^3 \frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \approx A\nu^3 e^{-\frac{B\nu}{T}}$$

zanedbáme pre $\nu \rightarrow \infty$