Elmag vlny

Maxwellove rovnice

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad = \rho_f + \rho_p$$

volný náboj ↗ ↖ polarizačný náboj

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\left.\begin{array}{l} \nabla \times \vec{B} = \mu_0 \vec{j} \quad \text{(neúplná)} \\ \nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t} \end{array}\right\} \text{rozpor:} \quad \nabla \cdot \nabla \times \vec{B} = 0 = \mu_0 \nabla \cdot \vec{j} \neq 0$$

⇓ doplnenie rovnice

$$\nabla \times \vec{B} = \mu_0 (\vec{j} + \vec{j}_M)$$

↖ Maxwellov posuvný prúd

$$\nabla \cdot \nabla \times \vec{B} = \mu_0 (\nabla \cdot \vec{j} + \nabla \cdot \vec{j}_M) = 0 \rightarrow \nabla \cdot \vec{j} = -\nabla j_M = \frac{\partial \rho}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t} \nabla \cdot \vec{E}$$

$$\Rightarrow j_M = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Vlnová rovnice – vakuum

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\cancel{\nabla \cdot \vec{E}}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\cancel{\mu_0 \vec{j}} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 0, \quad \vec{j} = 0$$

$$\Rightarrow \nabla^2 \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

3-rovnice:

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_i}{\partial t^2}$$

$\varepsilon_0 \mu_0 = \frac{1}{c^2}$ → rýchlosť svetla vo vakuu

Řešení vlnové rovnice

1D:

$$\frac{\partial^2 E_j}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \rightsquigarrow \left(\frac{\partial}{\partial z} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)E_j = 0$$

nové premenné: $\xi = ct - z$, $\tau = ct + z$

$$\frac{\partial}{\partial t} = c\frac{\partial}{\partial \xi} + c\frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial z} = -\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau}$$

→ dosadením: $\frac{\partial^2}{\partial \xi \partial \tau} E_j = 0 \Rightarrow E_j = f_1(\xi) + f_2(\tau)$

$$\Rightarrow E_j = f_1(ct - z) + f_2(ct + z) \rightarrow E_j = f(ct \pm z)$$

→ **Harmonická vlna:**

vlnové číslo

$$k = \frac{2\pi}{\lambda} \quad [m^{-1}]$$

$$\omega = \frac{2\pi}{T}$$

$$\vec{E} = \vec{E}_0 \cos(k(z - vt)) = \vec{E}_0 \cos(kz - kvt) = \vec{E}_0 \cos\left(kz - \frac{2\pi}{vT} vt\right) = \vec{E}_0 \cos\left(kz - \frac{2\pi}{T} t\right) =$$

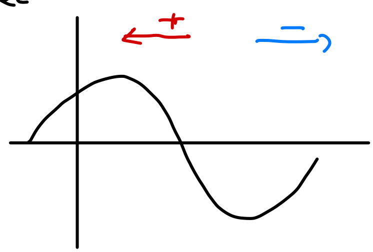
$$\vec{E} = \vec{E}_0 \cos(kz - \omega t) \xrightarrow{\text{komplexifikace}} \tilde{\vec{E}} = \mathrm{Re}\left\{\vec{E}_0 e^{i\overbrace{(kz - \omega t)}^{\phi \text{ - fáze}}}\right\}$$

$$\tilde{\vec{E}} = \vec{E}_0 e^{i(\underbrace{kz - \omega t}_{\text{fáze } \phi} \pm \delta)}$$

Re

$\pm\delta$ nábech fáze $- \phi(z=0, t=0)$

↳ posun vlny



3D:

$$\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t \pm \delta)$$

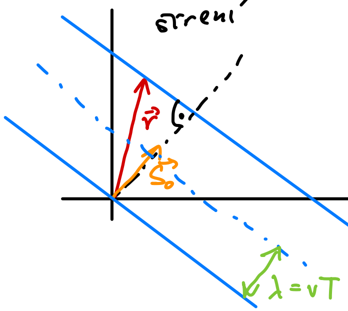
$$\tilde{\vec{E}} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t \pm \delta)} = \tilde{\vec{E}}_0\, e^{\pm i\delta}$$

Re

$\underbrace{\phi = \text{konst} \Leftrightarrow \text{rovina}}_{\text{vlnoplocha}}$

$$\vec{k} = \frac{2\pi}{\lambda}\vec{s}_0$$

$\vec{s}_0$ → smer šírenia



$\delta = 0,\ t = 0$

$\vec{k}\cdot\vec{r} = \text{konst}$

$\Downarrow$

$x s_1 + y s_2 + z s_3 = \underbrace{\vec{s}_0 \cdot \vec{r} = \text{konst}}_{\text{definice roviny}}$

kolmica na rovinu $\Rightarrow \vec{s}_0$

Vlnenie v dielek. prostredí

- pre rýchlosti elektrónu $v_e \ll c$ prevažuje elektrická interakcia s látkou $\Rightarrow \mu_r = 1$:

$$\vec{F} = \vec{F}_e + \vec{F}_B = q(\vec{E} + \vec{v}\times\vec{B}) \rightarrow |\vec{F}_E| = qE, \quad |\vec{F}_B|_{max} = |q v_e B| = \left|q \frac{v_e}{c} E\right|$$

$$\Rightarrow \frac{|F_B|_{max}}{|F_E|} = \frac{v_e}{c} \qquad B = \frac{E}{c}$$

$\mu_r = 1$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{B} = \mu_0\vec{j} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

$\vec{j} = \vec{j}_f + \vec{j}_p$ → polarizačný prúd

$\vec{P}$ … vektor polarizace

$$\nabla\times\nabla\times\vec{E} = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0\frac{\partial\vec{j}_f}{\partial t} - \mu_0\frac{\partial^2\vec{P}}{\partial t^2} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{j}_p = \frac{\partial\vec{P}}{\partial t}$$

$$\nabla\cdot\vec{P} = -\rho_p$$

$$\frac{\nabla\rho_f}{\varepsilon_0} + \frac{\nabla\rho_p}{\varepsilon_0} - \nabla^2\vec{E} = -\mu_0\frac{\partial\vec{j}_f}{\partial t} - \mu_0\frac{\partial^2\vec{P}}{\partial t^2} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla\cdot\vec{j}_p = \nabla\cdot\frac{\partial\vec{P}}{\partial t} = -\frac{\partial\rho_p}{\partial t}$$

$$\frac{\nabla\rho_f}{\varepsilon_0} + \frac{\nabla(\nabla\cdot\vec{P})}{\varepsilon_0} - \nabla^2\vec{E} = -\mu_0\frac{\partial\vec{j}_f}{\partial t} - \mu_0\frac{\partial^2\vec{P}}{\partial t^2}$$

→ úplná rovnice

Homogenní dielektrikum

$\nabla \cdot \vec{P} = 0 \qquad \rho_f = 0 \quad \vec{j}_f = 0$

$$\Rightarrow \nabla^2 \vec{E} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} + \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2}{\partial t^2} \overbrace{(\vec{P} + \varepsilon_0 \vec{E})}^{\vec{D}} \qquad \vec{P} = \varepsilon_0 \chi \vec{E}$$

$$\Rightarrow \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \underbrace{(1+\chi)}_{\varepsilon_r} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \underbrace{\varepsilon_0 \mu_0 \varepsilon_r}_{\frac{1}{v_2} = \varepsilon_0 \mu_0 \varepsilon_r} \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \rightarrow \text{liší se jen rychlostí}$$

index lomu $\quad \boxed{n = \frac{c}{v} = \sqrt{\varepsilon_r}} \geq 1$

$\lambda = vT = \frac{c}{n} T = \frac{\lambda_0}{n}$ ($\lambda_0 \rightarrow$ vakuum) $\quad \rightarrow \lambda$ se zkrátí v látce

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0} n = k_0 n \quad \rightarrow k$ se prodlouží v látce

Směr šíření

$f = f(\underbrace{\vec{s}_0 \cdot \vec{r} - vt}_{\xi}) \qquad \vec{s}_0 \vec{r} = s_{0x} x + s_{0y} y + s_{0z} z \qquad \vec{s}_0 = (s_{0x}, s_{0y}, s_{0z})$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} = s_{0x} \frac{\partial f}{\partial \xi} \qquad \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \dots, \dots \right) = \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial t} = -v \frac{\partial f}{\partial \xi} \qquad \left(s_{0y} \frac{\partial E_z}{\partial \xi} - s_{0z} \frac{\partial E_y}{\partial \xi}, \dots, \dots \right) = -\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \vec{s}_0 \times \frac{\partial}{\partial \xi} \vec{E}(\xi) = v \frac{\partial \vec{B}}{\partial \xi} \quad \Big/ \int d\xi \quad \Rightarrow \underbrace{\vec{s}_0 \times \vec{E} = v \vec{B}}_{\vec{E} \perp \vec{B}} + \text{konst}$

$E = vB$

z rovnice $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{s}_0 \times \vec{B} = -\frac{1}{v} \vec{E} \Rightarrow \vec{s}_0 \perp \vec{E}$

$\Rightarrow \vec{s}_0 \perp \vec{B} \perp \vec{E}$

→ separáciou premenných vlnovej rovnice: $E(r,t) = E_r(r)\, f(t)$
$= E_r(r)\, e^{-i\omega t}$

dostaneme Helmholtzovu rovnicu: $\nabla^2 E(r) + k^2 E(r) = 0$

Energie vlny

hustota energie

$$u_E = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 \qquad \vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$$

$$\cos(kz - \omega t)$$

MRV (monochrom. rovinná vlna)

$$u_E = \frac{1}{2}\varepsilon_0\varepsilon_r E_0^2 \cos^2(kz - \omega t)$$

optický obor – vysoké f ⇒ $\langle u_E\rangle_T = \frac{1}{2}\varepsilon_0\varepsilon_r E_0^2 \langle\cos^2(kz-\omega t)\rangle$

časová stredná hodnota

$$\langle\cos^2(kz-\omega t)\rangle = \frac{1}{T}\int_0^T \cos^2(kz-\omega t)\,dt \Rightarrow \langle u_E\rangle_T = \frac{1}{4}\varepsilon_0\varepsilon_r E_0^2$$

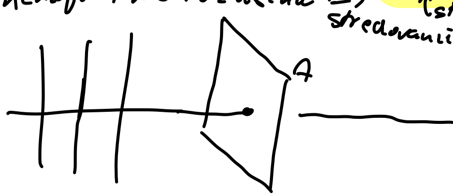
$$u_B = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}\frac{1}{v^2}E^2 = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 = u_E$$

$\Rightarrow \quad \langle u_B\rangle = \langle u_E\rangle \qquad u = u_E + u_B = \varepsilon_0\varepsilon_r E^2$

optické prístroje nemajú také rozlíšenie ⇒ stredovanie

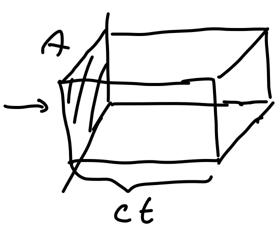
$$\langle u\rangle_T = \frac{1}{2}\varepsilon_0\varepsilon_r E_0^2 \qquad [u] = \mathrm{J\cdot m^{-3}}$$

stredná hustota energie



rýchlosť ... c

u ... u



$V = Act$ → objem

žiarivý výkon

$A = 1,\ t = 1$

$\langle uV\rangle_T \rightarrow \langle uc\rangle_T$

$\Rightarrow \langle uc\rangle_T = \langle\varepsilon_0 E^2 c\rangle$

Poyntingov vektor

souhlasné pro vlny

$$\vec{S} = \vec{E} \times \vec{H} \rightarrow S = EH = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

$[S] = W \cdot m^{-2}$

vakuum

$$\langle S \rangle_T = \left\langle \frac{E^2}{c\mu_0} \right\rangle = \langle \varepsilon_0 c E^2 \rangle = \langle u c \rangle = \frac{1}{2} \frac{E_0^2}{\mu_0 c} = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2$$

látka

$$S = EH = \varepsilon_0 c n E^2 \Rightarrow \langle S \rangle_T = \frac{1}{2} \varepsilon_0 c n E_0^2$$

$$I = \langle S \rangle_T$$ → intenzita vlny

Tlak světla

$\mathcal{E}_{foton} = h\nu$

$q_z = \frac{h\nu}{c}$ hybnost fotonu

$N_f \dots \frac{\#\ \text{fotonů (směr z)}}{m^2\ s}$

$S = N_f \mathcal{E}_{foton}$

$$G = \frac{N_f h\nu}{c} = N_f \cdot q$$

celková hybnost all foton

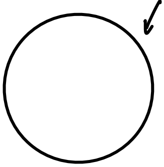
$\Delta G_z = \frac{N_f h\nu}{c} = \frac{S}{c} = P$ úplná absorpce

tlak

$\Delta G_z = \frac{2N_f h\nu}{c} = \frac{2S}{c} = P$ úplný odraz

Kulová vlna

vlnoplocha



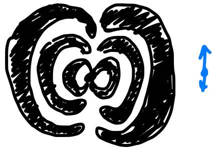
$$\psi(r,t) = \frac{A}{r} e^{i(kr \pm \omega t)} = \frac{f(r \pm vt)}{r}$$

→ kulová vlna

amplituda klesá

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Hertzov dipól



→ oscilující dipól

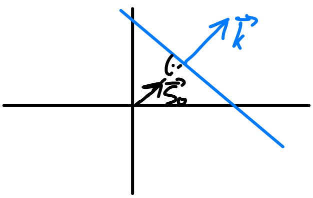
→ výkon dipólového žiarenia: $P_{tot} = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$, kde $p(t) = p_0 e^{-i\omega t}$ je oscilující dipól

Polarizace svetla

- popisuje $\vec{E}$

$\vec{E}, \vec{B} \perp \vec{k}$

$\vec{E} \perp \vec{B}$



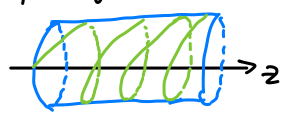
• pre homogénní, izotropní, dielektrikum neabsorbující: a_x, a_y amplitudy složek - reálne, nemenné

BUNO $\vec{s}_0 \parallel z$

$\vec{k} = (0, 0, k)$

$$E_x = a_x e^{i(kz - \omega t)}$$
$$E_y = a_y e^{i(kz - \omega t - \delta)}$$

} eliptický valec



→ vylúčenie času a z:

$$F(E_x, E_y) = \left(\frac{E_x}{a_x}\right)^2 - 2\left(\frac{E_x}{a_x}\right)\left(\frac{E_y}{a_y}\right)\cos\delta + \left(\frac{E_y}{a_y}\right)^2 - \sin^2\delta = 0$$

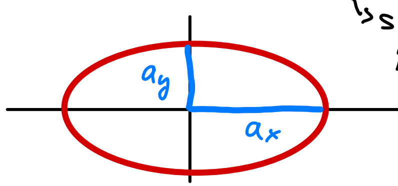
→ rovnice elipsy

pro $\delta = \pm \frac{\pi}{2}$

$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 = 1$$

→ elipsa, kde a_x, a_y sú poloosy

→ stratili sme pojem o čase, nevieme ktorým smerom sa točí



• speciální případy:

1) $\delta = 0$, $z = 0$

$E_x = a_x \cos\omega t$

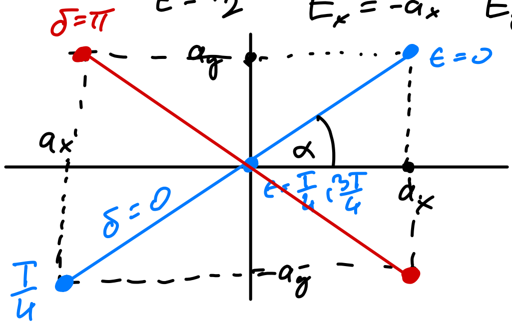
$E_y = a_y \cos\omega t$

pre $t = 0$ $E_x = a_x$, $E_y = a_y$

$t = \frac{T}{4}$ $E_x = E_y = 0$

$t = \frac{T}{2}$ $E_x = -a_x$ $E_y = -a_y$

2) $\delta = \pi$



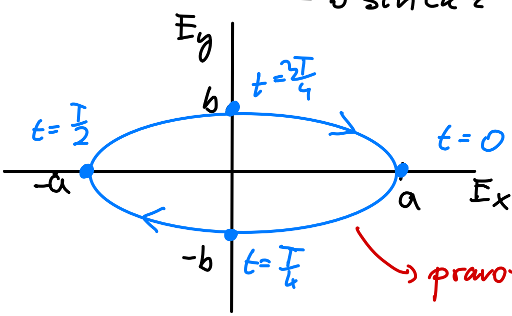
$tg\,\alpha = \frac{a_y}{a_x}$

3) $\delta = \frac{\pi}{2}$

$E_x = a e^{i(kz-\omega t)}$ $E_y = b e^{i(kz-\omega t - \frac{\pi}{2})}$

$E_x = a\cos(kz-\omega t)$ $z=0 \Rightarrow$ $E_x = a\cos(\omega t)$

$$E_y = Re\{b e^{-i\frac{\pi}{2}} e^{i(kz-\omega t)}\}$$
$$= Re\{-ib\, e^{i(kz-\omega t)}\}$$
$$= b\sin(kz-\omega t) \quad z=0 \Rightarrow E_y = -b\sin(\omega t)$$



$t = \frac{T}{4}$: $E_x = 0$ $E_y = -b$

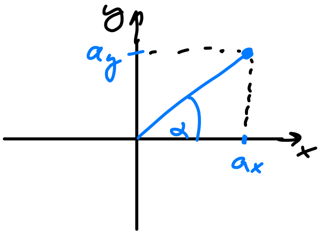
pravotočivé eliptický polarizované světlo

• pro $a_x = a_y$ dostaneme kruhovo polarizované světlo

Jonesův formalizmus

- světlo popsané Jonesovými vektory
- polarizační prvky – Jonesovy matice

RCP – right circular polarized
LEP – left eliptical polarized



$$\operatorname{tg}\alpha = \frac{a_y}{a_x}$$

$$\sin\alpha = \frac{a_y}{\sqrt{a_x^2+a_y^2}} \qquad \cos\alpha = \frac{a_x}{\sqrt{a_x^2+a_y^2}}$$

$$E_x = a_x e^{i\varphi} \qquad\qquad E_x = \sqrt{a_x^2+a_y^2}\cos\alpha\, e^{i\varphi} = \frac{a_y}{\sin\alpha}\cos\alpha\, e^{i\varphi}$$
$$E_y = a_y e^{i(\varphi-\delta)} \quad\rightarrow\quad E_y = \frac{a_y}{\sin\alpha}\sin\alpha\, e^{i(\varphi-\delta)}$$

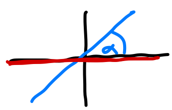
$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \underbrace{\frac{a_y}{\sin\alpha}}_{E_{ef}} \underbrace{\begin{pmatrix} \cos\alpha \\ \sin\alpha\, e^{-i\delta} \end{pmatrix}}_{\text{Jonesov vektor}} e^{i\varphi} \qquad \vec{J} = \begin{pmatrix} \cos\alpha \\ \sin\alpha\, e^{-i\delta} \end{pmatrix}$$

$$\vec{J}\,\vec{J}^{*} = \cos^2\alpha + \sin^2\alpha = 1 \quad\Rightarrow\quad \text{jednotkový vektor}$$

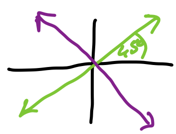
- speciální případy:

1) $\delta = 0$

$$\vec{J} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$



- $\alpha = 0$ $\quad \vec{J} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- $\alpha = \frac{\pi}{4}$ $\quad \vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- $\alpha = -\frac{\pi}{4}$ $\quad \vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$



2) $\delta = \frac{\pi}{2}$

$$\vec{J} = \begin{pmatrix} \cos\alpha \\ \sin\alpha\, e^{-i\frac{\pi}{2}} \end{pmatrix} \xrightarrow[\substack{\alpha=\frac{\pi}{4} \\ a_x = a_y}]{} \vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \vec{J}_{RCP}$$

$3)\ \delta = -\frac{\pi}{2}$ $\quad \vec{J} = \begin{pmatrix}\cos\alpha \\ \sin\alpha \, e^{i\frac{\pi}{2}}\end{pmatrix} \rightarrow \quad \vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ i\end{pmatrix} = \vec{J}_{LCP}$

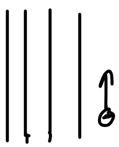
$$E_{ef} = \sqrt{a_x^2 + a_y^2} = \frac{a_y}{\sin\alpha} = \frac{a_x}{\cos\alpha}$$

$$\langle u_E \rangle_T = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(a_x^2 + a_y^2\right) = \frac{1}{2}\varepsilon_0 E_{ef}^2$$

Príprava polarizovaného svetla

Polarizátor

- asymetrická absorbce



pohyb elektronů v drátkách => Joulovo teplo => ztráta energie v tomto smere

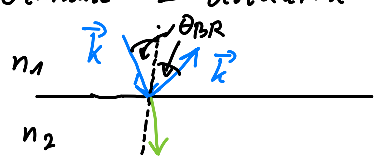
kovové drátky

-> polarizácia v kolmém smeru na drátky -> kmitosmer

- molekuly tvaru
- ztráta 50% intenzity

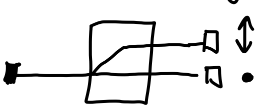
Polarizace odrazem

• rozhraní 2 dielektrik



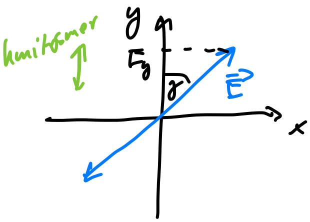
• pri odraze pod uhlom $\theta_{BR} = tg\frac{n_2}{n_1}$ dochádza k polarizácii v rovine rozhrania

Polarizace dvojlomem



polarizace kolmo na smer šírenia

Polarizátor



IN: $\vec{E} = E_0 \cos(\varphi_\gamma - \omega_\gamma t)$

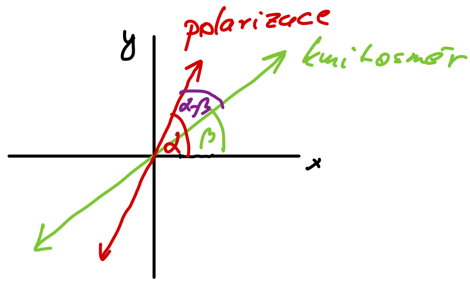
OUT: $E_y = E_0 \cos\gamma \cos(\varphi_\gamma - \omega_\gamma t)$

$$I_\gamma = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0 E_0^2 \cos^2\gamma = I_0 \cos^2\gamma$$

$\boxed{I_\gamma = I_0 \cos^2\gamma}$ Malusův zákon → pre polarizované

$$I_{pol} = \frac{1}{2\pi}\int_0^{2\pi} I_0 \cos^2\gamma \, d\gamma = \frac{I_0}{2} \quad \rightarrow \text{pre nepolarizované}$$

Pomocou Jonesa:



IN: $E = E_0 \begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix} e^{i\varphi}$

OUT: $E = E_0 \cos(\alpha-\beta) \begin{pmatrix}\cos\beta \\ \sin\beta\end{pmatrix} e^{i\varphi}$

$= E_0 (\cos\alpha\cos\beta + \sin\alpha\sin\beta)\begin{pmatrix}\cos\beta \\ \sin\beta\end{pmatrix} e^{i\varphi} =$

$= E_0 \begin{pmatrix}\cos^2\beta\cos\alpha + \sin\alpha\sin\beta\cos\beta \\ \cos\alpha\cos\beta\sin\beta + \sin\alpha\sin^2\beta\end{pmatrix} e^{i\varphi} =$

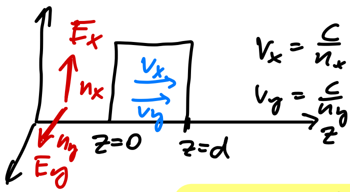
$$= E_0 \underbrace{\begin{pmatrix}\cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta\end{pmatrix}}_{\overleftrightarrow{T}_{pol}} \begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix} e^{i\varphi} =$$

$$\overleftrightarrow{T}_{pol} = \begin{pmatrix}\cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta\end{pmatrix}$$

→ ide o maticu projekcie do kmitosmeru

$v = \begin{pmatrix}\cos\beta \\ \sin\beta\end{pmatrix}$ je kmitosmer $\Rightarrow P_v = \begin{pmatrix}\cos\beta \\ \sin\beta\end{pmatrix}(\cos\beta \;\; \sin\beta) = T_{pol}$

Fázová doštička



$v_x = \frac{c}{n_x}$

$v_y = \frac{c}{n_y}$

→ rýchlosť svetla závisí od polarizácie:
v_x je rýchlosť v smere polarizácie x
v_y je rýchlosť v smere polarizácie y

→ zmena fáze doštičkou

IN: $\vec{E}_{in} = \begin{pmatrix} E_{x0} \\ E_{y0} \end{pmatrix} e^{i\varphi}$

$E_{x0} = a_x$

$E_{y0} = a_y e^{-i\delta}$

pre $z=0$:
$\vec{E}_{in} = \begin{pmatrix} E_{x0} \\ E_{y0} \end{pmatrix} e^{-i\omega t}$

OUT: $z=d$

$E_x(z=d) = E_{0x} e^{ik_0 n_x d} e^{-i\omega t}$

$E_y(z=d) = E_{0y} e^{ik_0 n_y d} e^{-i\omega t}$

$$\vec{E}_{out} = \underbrace{\begin{pmatrix} e^{ik_0 n_x d} & 0 \\ 0 & e^{ik_0 n_y d} \end{pmatrix}}_{T_y} \underbrace{\begin{pmatrix} \hat{E}_{0x} \\ \hat{E}_{0y} \end{pmatrix}}_{\vec{E}_{IN}(z=0)} e^{-i\omega t}$$

$$T_y = \begin{pmatrix} e^{i\varphi_x} & 0 \\ 0 & e^{i\varphi_y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\varphi_y - \varphi_x)} \end{pmatrix} \underbrace{e^{i\varphi_x}}_{=1} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$\varphi_x = k_0 n_x d$

$\varphi_y = k_0 n_y d$

$\varphi = k_0 d (n_y - n_x)$

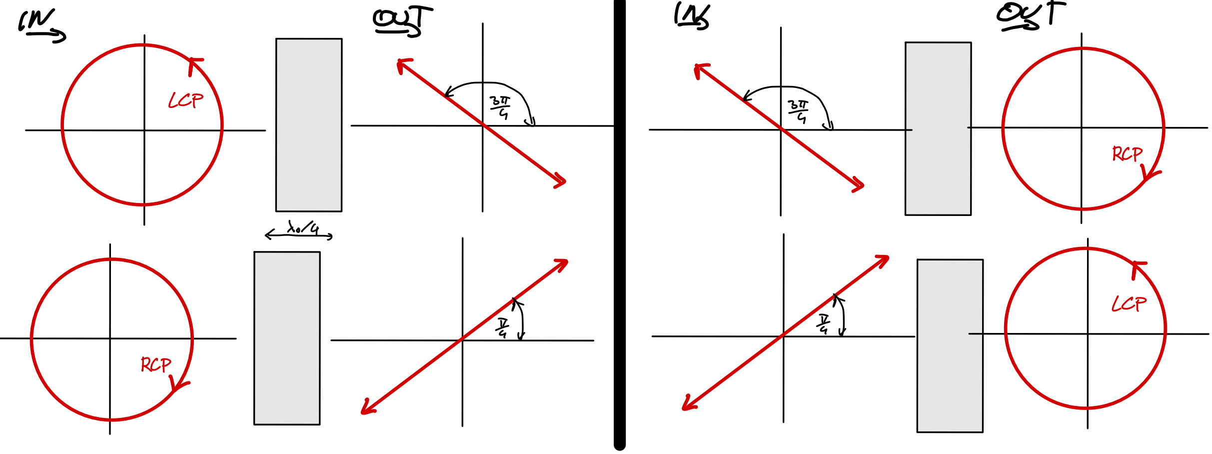
nd ... optická dráha

- špec. prípady

$\varphi = \frac{\pi}{2}$ $\quad \frac{\pi}{2} = (n_y - n_x) \frac{2\pi}{\lambda_0} d \Rightarrow (n_y - n_x) d = \frac{\lambda_0}{4}$ → štvrťvlnová doštička

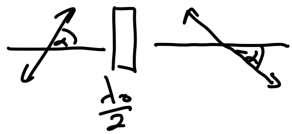
$T_{\varphi = \frac{\pi}{2}} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ $\quad J_{IN} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \longrightarrow J_{out} = T_{\frac{\pi}{2}} J_{out} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

RCP ◯ □ $\frac{\lambda_0}{4}$ LCP ◯ $\quad$ LCP $\quad$ lineárne

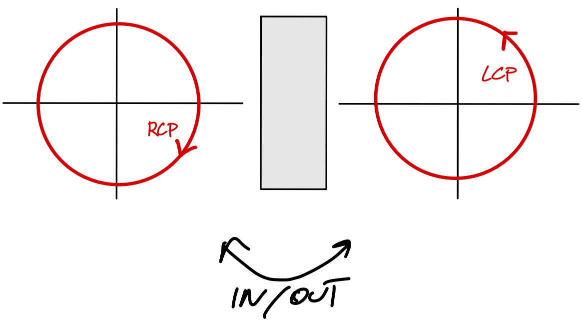


$\varphi = \pi \qquad (n_y - n_x)\,d = \frac{\lambda_0}{2} \rightarrow$ půlvlnová destička

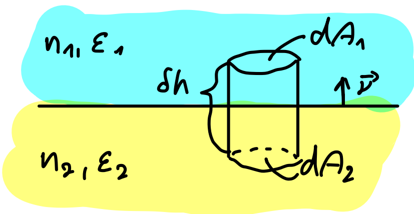
$$T_\pi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$J_{in} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \qquad J_{out} = \begin{pmatrix} \cos\alpha \\ -\sin\alpha \end{pmatrix}$$



Odraz a lom na rozhraní dielektrik



Podmínky na rozhraní:
$\vec{E}, \vec{B}, \vec{D}, \vec{H}$

Normálové složky:

$\nabla \cdot \vec{B} = 0$ Gauss

$$\int_V \nabla \cdot \vec{B} = \oint_{\delta V} \vec{B} \cdot d\vec{S} = \oint_{\delta V} \vec{B} \cdot \vec{\nu}_s \, dS = \int \vec{B}_1 \vec{\nu} dA_1 - \vec{B}_2 \vec{\nu} dA_2 = 0$$

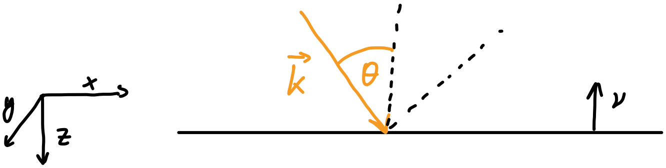
$$\Rightarrow \quad \vec{B}_1 \cdot \vec{\nu} = \vec{B}_2 \cdot \vec{\nu} \quad \Rightarrow \quad B_n^1 = B_n^2$$

$$\nabla \cdot \vec{D} = \rho \quad , \text{pro } \rho = 0 \quad \dots \quad D_n^1 = D_n^2$$

Tečné složky

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

$$E_t^1 = E_t^2 \qquad H_t^1 = H_t^2$$



rovina rozhraní $\quad \vec{\nu} = (x, y, 0)$

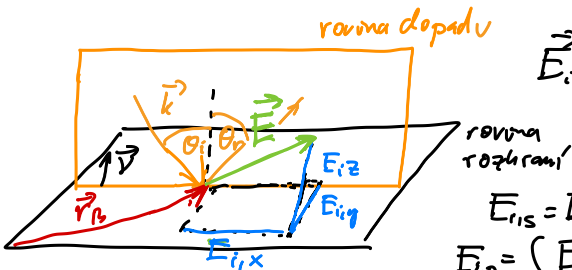
rovina dopadu $\quad \vec{k}, \vec{\nu}$

$(x, 0, z)$

přicházející ..... $\vec{E}_i = \vec{E}_{0i} e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)}$

odrazená ..... $\vec{E}_r = \vec{E}_{0r} e^{i(\vec{k}_r \cdot \vec{r} - \omega_r t)}$

propustená ..... $\vec{E}_t = \vec{E}_{0t} e^{i(\vec{k}_t \vec{r} - \omega_t t)}$



$\vec{E}_{i,t} = (E_{ix}, E_{iy}, 0)$

$E_{i,s} = E_{i,y}$ $\perp$ rovina dopadu

$E_{i,p} = (E_{i,x}, 0, E_{i,z})$ // rovina dopadu

$\vec{E}_{i,\text{tečná}} + \vec{E}_{r,\text{tečná}} = \vec{E}_{t,\text{tečná}} \Rightarrow \vec{k}_i \cdot \vec{r}_B - \omega_i t = \vec{k}_r \vec{r}_B - \omega_r t = \vec{k}_t \vec{r}_B - \omega_t t$

$\Rightarrow$ $\vec{r} = 0$: $\omega_i = \omega_r = \omega_t$ → frekvence se nemení

$t = 0$: $\vec{k}_i \cdot \vec{r}_B = \vec{k}_r \cdot \vec{r}_B = \vec{k}_t \cdot \vec{r}_B$ $\quad \vec{k}_i = (k_{ix}, 0, k_{iz})$, $\vec{r}_B = (x, y, 0)$

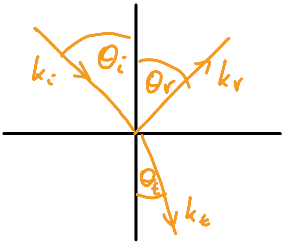
$\vec{k}_i \cdot \vec{r}_B = k_{ix} \cdot x = k_{rx} x + k_{ry} y = k_{tx} x + k_{ty} y$

→ má to platit pre $\forall x, y$:

–

$k_{ix} = k_{rx} = k_{tx}$; $k_{ry} = k_{ty} = 0$

$$\left.\begin{aligned} \vec{k}_r &= (k_{rx}, 0, k_{rz}) \\ \vec{k}_t &= (k_{tx}, 0, k_{tz}) \\ \vec{k}_i &= (k_{ix}, 0, k_{iz}) \end{aligned}\right\}$$ leží všechny v rovine dopadu



$k_{ix} = k_{rx} \rightarrow \cancel{\frac{\omega}{c}}\, \cancel{n_1} \sin\theta_i = \cancel{\frac{\omega}{c}}\, \cancel{n_1} \sin\theta_r$

$\Rightarrow \quad \theta_i = \theta_r$ → zákon odrazu

$k_{ix} = k_{tx} \rightarrow \cancel{\frac{\omega}{c}}\, n_1 \sin\theta_i = \cancel{\frac{\omega}{c}}\, n_2 \sin\theta_t$

$\Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t$

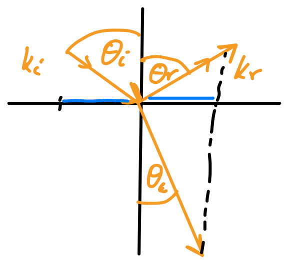
↳ zákon lomu

$\theta_i = \theta_r$ – zákon odrazu

$n_1 \sin\theta_i = n_2 \sin\theta_t$ – zákon lomu

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2}$$

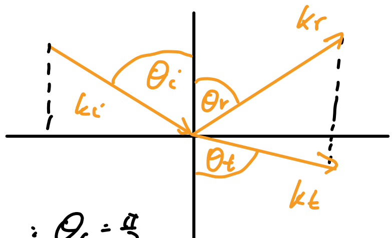
1) $n_1 < n_2 \Rightarrow \theta_t < \theta_i$



$\theta_i = 0 \Rightarrow$ kolmý dopad

$\theta_t = \theta_r = 0$

2) $n_1 > n_2 \quad \theta_t > \theta_i$



$\theta_t = \frac{\pi}{2}$ → max. úhel lomu

kritický úhel dopadu θ_c : $\theta_t = \frac{\pi}{2}$

⇓

$n_1 \sin\theta_c = n_2 \sin\frac{\pi}{2} = n_2 \Rightarrow \sin\theta_c = \frac{n_2}{n_1}$

pro $\sin\theta_c > \frac{n_2}{n_1}$ ⇒ úplný odraz

Fresnelovy vztahy

s- polarizace (kolmo na rovinu dopadu)

$$\vec{E}_i = (0, E_{iy}, 0) \qquad \vec{k}_i = (k_{ix}, 0, k_{iz})$$

$$E_{iy} + E_{ry} = E_{ty}$$

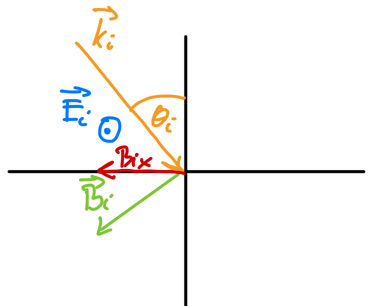
$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{B}_i = \frac{1}{\omega}(\vec{k}_i \times \vec{E}_i) = \frac{1}{\omega}(-k_{iz} E_{iy}, 0; k_{ix} E_{iy})$$

$$\vec{B}_i = E_{iy} \frac{n_1}{c}(-\cos\theta_i, 0, \sin\theta_i)$$

$$\vec{B}_t = E_{ty} \frac{n_2}{c}(-\cos\theta_t, 0, \sin\theta_t)$$

$$\vec{B}_r = E_{ry} \frac{n_1}{c}(\cos\theta_i, 0, \sin\theta_i)$$



podm: $B_{ix} + B_{rx} = B_{tx}$

$\downarrow$

$$-E_{iy} \frac{n_1}{c} \cos\theta_i + E_{ry} \frac{n_1}{c} \cos\theta_i = -E_{ty} \frac{n_2}{c} \cos\theta_t$$

$$-E_{iy} n_1 \cos\theta_i + E_{ry} n_1 \cos\theta_i = -(E_{iy} + E_{ry}) n_2 \cos\theta_t$$

$r_s = \frac{E_{ry}}{E_{iy}}$ → amplitudový koef. odrazu

$t_s = \frac{E_{ty}}{E_{iy}}$ → amplitudový koef. lomu

$$-n_1 \cos\theta_i + r_s n_1 \cos\theta_i = -n_2 \cos\theta_t - r_s n_2 \cos\theta_t$$

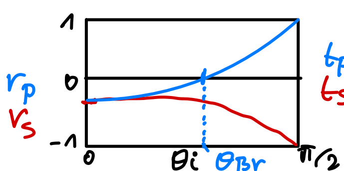
$$\boxed{r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad t_s = r_s + 1 = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}}$$

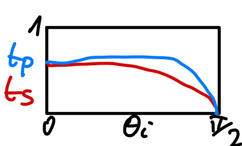
1) $n_1 < n_2$

$\theta_i = 0 \Rightarrow \theta_t = 0 \quad r_s = \frac{n_1 - n_2}{n_1 + n_2}$ i pro vzduch ($n_1 = 1$), sklo ($n_2 = 1,5$)

$\Rightarrow r_s = -0,2$

$\theta_i = \frac{\pi}{2} \longrightarrow r_s = -1$





p-polarizace (// na rovinu dopadu)

$$\boxed{r_p = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}} \quad \boxed{t_p = (1+r_p)\frac{\cos\theta_i}{\cos\theta_t}} \quad r_p = \frac{tg(\theta_t - \theta_i)}{tg(\theta_t+\theta_i)}$$

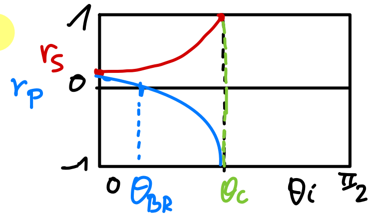
→ pro $\theta_i = \theta_{Br}$ je $r_p = 0$ ⇒ světlo je úplně polarizované v s-směru

$$\boxed{tg\,\theta_{Br} = \frac{n_2}{n_1}}$$ → Brewsterův zákon

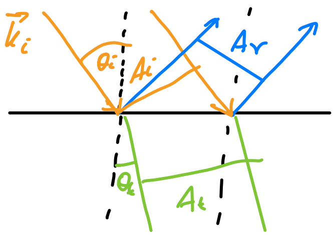
$$\boxed{r_s = \frac{n_1^2 - n_2^2}{n_1^2+n_2^2}}$$

odraz a lom $n_1 > n_2$

$\theta_{max} = \theta_c$



Intenzity



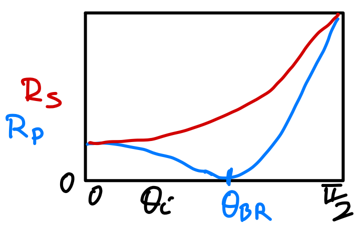
$$A = \frac{A_i}{\cos\theta_i} = \frac{A_r}{\cos\theta_r} \rightarrow A_i = A_r$$

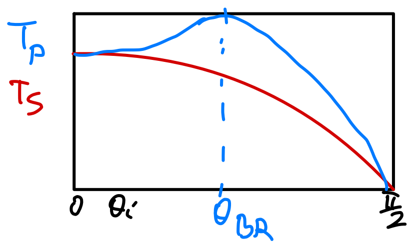
$$A = \frac{A_t}{\cos\theta_t}$$

$$\underbrace{\tfrac{1}{2}\varepsilon_0 c n_1}_{J_i} E_{0i}^2 \underbrace{A\cos\theta_i}_{A_i} = \underbrace{\tfrac{1}{2}\varepsilon_0 c n_1 E_{0r}^2}_{J_r} \underbrace{A\cos\theta_i}_{A_r} + \underbrace{\tfrac{1}{2}\varepsilon_0 c n_2 E_{0t}^2}_{J_t} \underbrace{A\cos\theta_t}_{A_t}$$

$$\boxed{R_{s,p} = \frac{J_r}{J_i} = \frac{E_{0r}^2}{E_{0i}^2} = r_{s,p}^2}$$

$$\boxed{T_{s,p} = \frac{J_t}{J_i} = \frac{n_2}{n_1}\frac{E_{0t}^2}{E_{0i}^2}\frac{\cos\theta_t}{\cos\theta_i} = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}|t_{s,p}|^2}$$





Interference

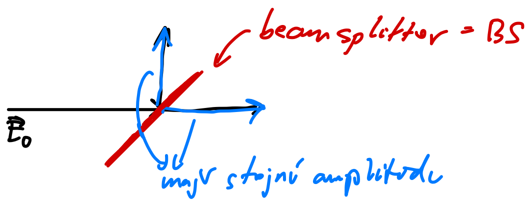
Skládání vln stejné frekvence

Princip superpozice vln... $\sum_n \vec{E}_n = \vec{E}(\vec{r},t)$

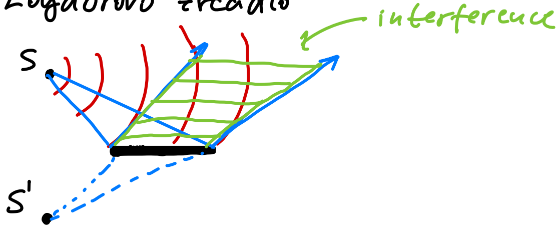
→ intenzita není součtem od vln: $I(\vec{r},t) \cancel{=} \sum_n I_n$

Vlny stejné frekvence
- amplituda (dělení)
- vlnoplocha

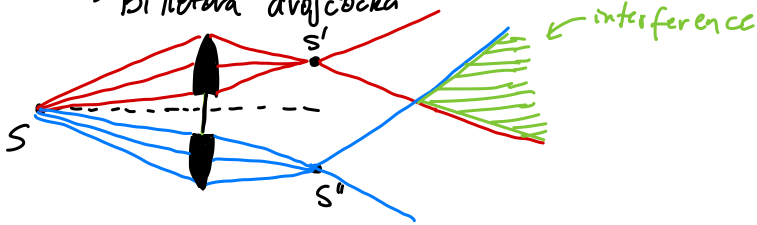
Dělení amplitudy ... dělič svazku



Dělení vlnoplochy → Loydovo zrcadlo



→ Billetova dvojčočka



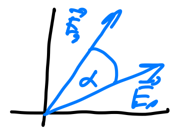
Skládaní 2 lin. polarizovaných monochrom. vln

$$\vec{E}_1 = \vec{E}_{01}\, e^{i(\vec{k}_1\cdot\vec{r} - \omega t + \delta_{01})}$$
$$\vec{E}_2 = \vec{E}_{02}\, e^{i(\vec{k}_2\cdot\vec{r} - \omega t + \delta_{02})}$$

$$|\vec{k}_1| = |\vec{k}_2|$$
$$\varphi_i = \vec{k}_i\cdot\vec{r} - \omega t + \delta_{0i}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = \frac{1}{4}\varepsilon_0 n^2 \vec{E}\vec{E}^* =$$
$$= \frac{1}{4}\varepsilon_0 n^2 (\vec{E}_1 + \vec{E}_2)(\vec{E}_1^* + \vec{E}_2^*) =$$
$$= \frac{1}{4}\varepsilon_0 n^2 \left(E_{01}^2 + E_{02}^2 + \vec{E}_{01}\cdot\vec{E}_{02}\, e^{i(\varphi_1 - \varphi_2)} + \vec{E}_{01}\cdot\vec{E}_{02}\, e^{-i(\varphi_1 - \varphi_2)}\right)$$
$$= I_1 + I_2 + \frac{1}{2}\varepsilon_0 n^2 \vec{E}_{01}\cdot\vec{E}_{02}\cos(\underbrace{\varphi_1 - \varphi_2}_{\delta_{12}}) = I_1 + I_2 + \frac{1}{2}\varepsilon_0 n^2 E_{01}E_{02}\cos\alpha\cos\delta_{12}$$



→ dálej budeme predpokladat' $\underline{\cos\alpha = 1}$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta_{12}(\vec{r})}$$

→ pokud $I_1 = I_2 = I_0$: $I = 2I_0(1 + \cos\delta_{12}(\vec{r}))$

$$I = 4 I_0 \cos^2\left(\frac{\delta_{12}(\vec{r})}{2}\right)$$

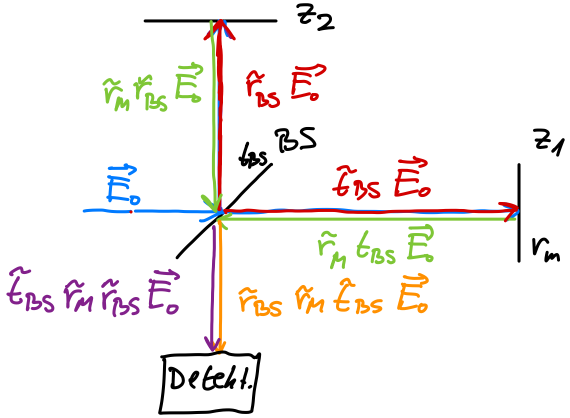
$$\boxed{\begin{aligned} I_{max} &= I_1 + I_2 + 2\sqrt{I_1 I_2} \\ I_{min} &= I_1 + I_2 - 2\sqrt{I_1 I_2} \end{aligned}}$$

$$\boxed{V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}} = \frac{4\sqrt{I_1 I_2}}{2I_1 + 2I_2} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \overset{I_1 = I_2}{=} 1$$

viditelnost IF obrazce

$$\delta_{12}(\vec{r}) = (\vec{k}_1 - \vec{k}_2)\cdot\vec{r} + \delta_{01} - \delta_{02}$$

Michelsonův interferometr



$$\left.\begin{aligned} E_{01} &= \tilde{r}_{BS}\, \tilde{r}_M\, t_{BS}\, E_0 \\ E_{02} &= \tilde{t}_{BS}\, \tilde{r}_M\, \tilde{r}_{BS} \end{aligned}\right\} \to E_{01} = E_{02} \Rightarrow I_1 = I_2$$

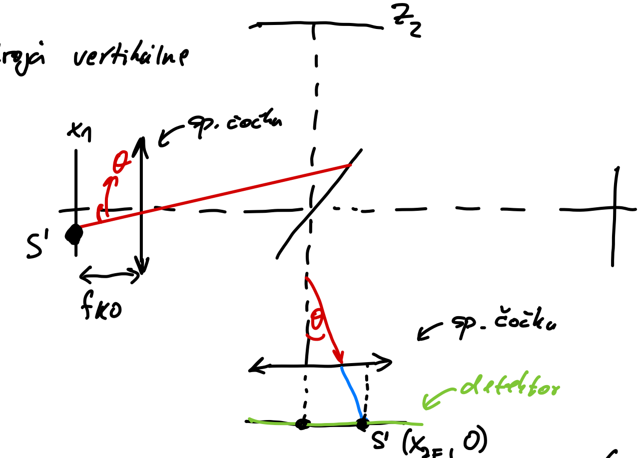
$$\delta_{12} = k_0 n_0 \Delta_0 \quad ; \quad \Delta_0 = 2(d_1 - d_2) \quad \Rightarrow \delta_{12} \neq f(\vec{r}) \Rightarrow$$ nebude interferenčný obrazec

$$\delta_{12} = k_0 n_0\, 2(d_1 - d_2) = \frac{4\pi}{\lambda_0} n (d_1 - d_2)$$

$$\to \frac{4\pi}{\lambda}(d_1 - d_2) = 2m\pi \quad \to MAX$$

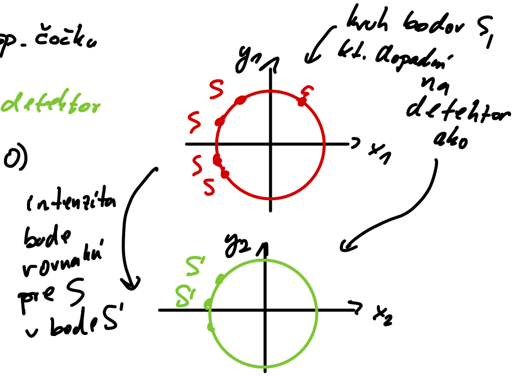
$$\to \frac{4\pi}{\lambda}(d_1 - d_2) = (2m-1)\pi \quad \to MIN$$

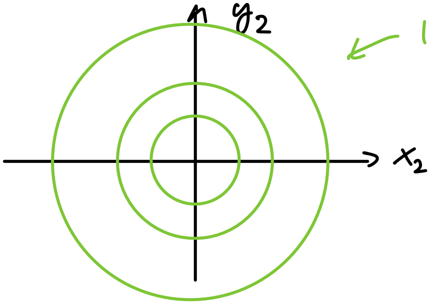
→ posúvanie zdroja vertikálne



$$tg\,\theta = \frac{x_{1F}}{f_{K0}}$$

$$\delta_{12} = \frac{4\pi}{\lambda}(d_1 - d_2)\cos\theta$$

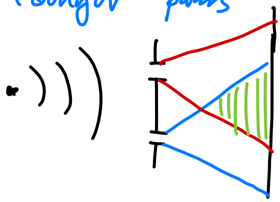




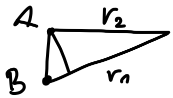
IF kroužky stejné intenzity – Haidingerovy kroužky } řídicí parametr je θ

$$\operatorname{tg}\theta = \frac{\sqrt{x_{1F}^2 + y_{1F}^2}}{f_{ko}} = \frac{\sqrt{x_{2F}^2 + y_{2F}^2}}{f_{ko}}$$

Youngův pokus

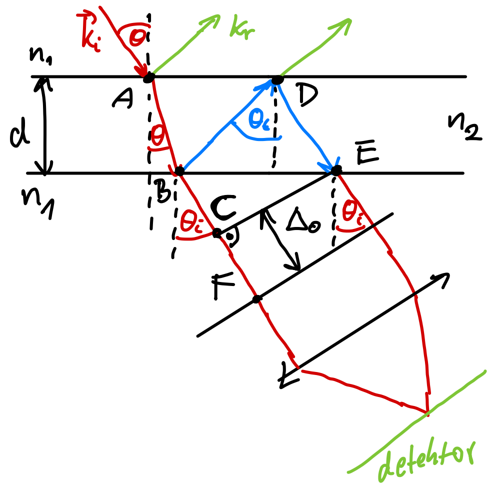


MATNICE
aka Foťák



$$\Delta_0 = r_2 - r_1 = konst$$

IF NA PLANPARALELNÍ DESCE



$$\overline{BDE} = \frac{2d}{\cos\theta_t}$$

$$v_2 = \frac{c}{n_2} \qquad t = \frac{2dn_2}{\cos\theta_t\, c}$$

$$\overline{BF} = \frac{c}{n_1}\cdot\frac{2dn_2}{\cos\theta_t c} = \frac{2d}{\cos\theta_t}\frac{n_2}{n_1}$$

$$\overline{CF} = \overline{BF} - \overline{BC}$$

$$\sin\theta_i = \frac{\overline{BC}}{\overline{BF}} \qquad \overline{BE} = 2d\operatorname{tg}\theta_t$$

$$\downarrow$$

$$\overline{BC} = \overline{BF}\sin\theta_i$$

$$\overline{CF} = \frac{2dn_2}{\cos\theta_t n_1} - 2d\operatorname{tg}\theta_t\sin\theta_i$$

Fázový rozdíl $\delta_{12} = k_0 n_1 \overline{CF} = \frac{2\pi}{\lambda_0} n_1\left(\frac{2d}{\cos\theta_t}\frac{n_2}{n_1} - 2d\operatorname{tg}\theta_t\sin\theta_i\right) =$

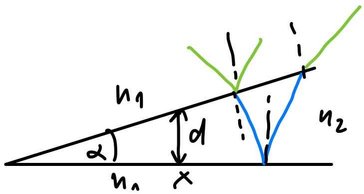
$$= \frac{4\pi}{\lambda_0}\frac{dn_2}{\cos\theta_t}\left(1 - \frac{n_1\sin\theta_t\sin\theta_i}{n_2}\right) =$$

$$= \frac{4\pi}{\lambda_0}\frac{dn_2}{\cos\theta_t}\left(1 - \sin^2\theta_t\right) = \frac{4\pi}{\lambda_0} n_2 d\cos\theta_t$$

$$\delta_{12} = \frac{4\pi}{\lambda_2} d\cos\theta_t \;\to\; \frac{4\pi}{\lambda_2} d\cos\theta_t = 2m\pi \qquad \text{MAX}$$

$$\to\; \frac{4\pi}{\lambda_2} d\cos\theta_t = (2m-1)\pi \qquad \text{MIN}$$

$$\delta_{12} = \frac{4\pi}{\lambda_2} d\cos\theta_t + \pi$$



$$\operatorname{tg}\alpha = \frac{d}{x} \approx \alpha \qquad d = x\alpha$$

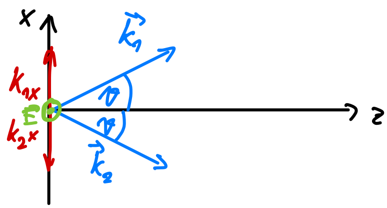
$$\frac{4\pi}{\lambda_2} d\cos\theta_t + \pi = 2m\pi \qquad \text{MAX}$$

$$d_{max} = \left(m - \frac{1}{2}\right)\frac{\lambda_2}{2\cos\theta_t}$$

$$x_{max} = \left(m - \frac{1}{2}\right)\frac{\lambda_2}{2}\,\frac{1}{\alpha\cos\theta_t}$$

Dvojsvazková interference

• $\vec{k}_1 \neq \vec{k}_2 \quad |\vec{k}_1| = |\vec{k}_2|$



$$\vec{k}_1 = (k_{1x}, 0, k_{1z}) \qquad k_{1x} = k\sin\vartheta$$

$$\vec{k}_2 = (k_{2x}, 0, k_{2x}) \qquad k_{2x} = -k\sin\vartheta$$

$$k_{1z} = k_{2z} = k\cos\vartheta =: k_z$$

$$k_{1x} = -k_{2x} =: k_x$$

$$\vec{E}_{1,2} = (0, E_{1,2y}, 0)$$

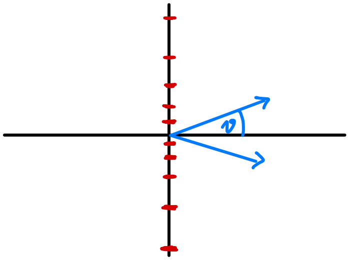
$$E = E_{1y} + E_{2y} = E_0 e^{i(\overbrace{\vec{k}_1\cdot\vec{r}}^{=k_x x + k_z z} - \omega t + \frac{\delta_0}{2})} + E_0 e^{i(\overbrace{\vec{k}_2\cdot\vec{r}}^{=-k_x x + k_z z} - \omega t - \frac{\delta_0}{2})} =$$

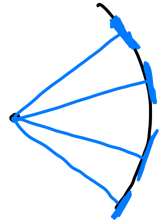
$$= E_0 e^{i(k_z z - \omega t)}\left(e^{i(k_x x + \frac{\delta_0}{2})} + e^{-i(k_x x + \frac{\delta_0}{2})}\right) = 2E_0\cos\left(k_x x + \frac{\delta_0}{2}\right)e^{i(k_z z - \omega t)}$$

$$I = \frac{1}{4}\varepsilon_0 n^2 E E^* = \underbrace{\frac{1}{4}\varepsilon_0 n^2 E_0^2}_{I_0} 4\cos^2\left(k_x x + \frac{\delta_0}{2}\right)$$

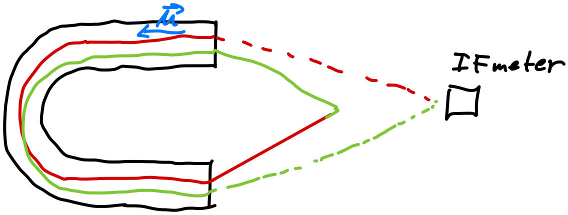
$$I = 4I_0\cos^2\left(k_x x + \frac{\delta_0}{2}\right) = 2I_0\left(1 + \cos(2k_x x + \delta_0)\right)$$

$$I_{max}: \; 2k_x x + \delta_0 = 2m\pi \xrightarrow{\delta_0 = 0} \frac{\overset{2}{\cancel{4}}\pi}{\lambda_0} n\sin\vartheta\, x = \cancel{2}m\pi \to \boxed{x_{max} = \frac{m\lambda_0}{2n\sin\vartheta} = \frac{m\lambda}{2\sin\vartheta}}$$





aproximace kulové vlny pomocou rovinných
→ výsledná gulová je IF rovinných

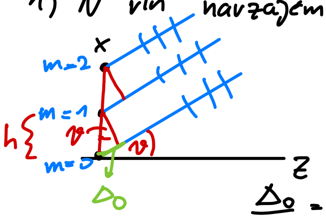


$$\frac{c}{n} + \underbrace{(1-n^2)\,u}_{\text{strhávání éteru}}$$

⇒ neskor vysvetlené relativistickým skladaním rýchlostí

Skládání mnoha vln stejných frekvencí

1) N vln navzájem fázově posunutých o δ



$$\frac{\Delta_0}{h} = \sin\vartheta \quad \rightarrow \quad \delta = k_0 n \Delta_0 = \frac{2\pi}{\lambda_0} n h \sin\vartheta$$

$m = 0, \dots, N-1$

$$E_m = \underbrace{E_0 e^{i(k\sin\vartheta\, x + k\cos\vartheta\, z)} e^{-i\omega t}}_{E_{m=0}} e^{im\delta} = E_{m=0}\, e^{im\delta}$$

$$E_{tot}(\vec{r}) = \sum_{m=0}^{N-1} \tilde{E}_{m=0}\, e^{im\delta} = \tilde{E}_{m=0} \frac{e^{iN\delta} - 1}{e^{i\delta} - 1}$$

$$I(\vec{r}) = \frac{1}{4}\varepsilon_0 n^2 \tilde{E}_{tot} \tilde{E}^*_{tot} = \frac{1}{4}\varepsilon_0 n^2 E_0^2 \underbrace{\frac{e^{iN\delta}-1}{e^{i\delta}-1} \cdot \frac{e^{-iN\delta}-1}{e^{-i\delta}-1}}_{1 - e^{-iN\delta} - e^{iN\delta} + 1 \,=\, 2 - 2\cos N\delta} = \frac{1}{4}\varepsilon_0 n^2 E_0^2 \frac{1-\cos N\delta}{1-\cos\delta}$$

$$I(\vec{r}) = \frac{1}{4}\varepsilon_0 n^2 E_0^2 \frac{1-\cos N\delta}{1-\cos\delta} = I_0 \frac{\sin^2 \frac{N\delta}{2}}{\sin^2 \frac{\delta}{2}} = I_0 N^2 \frac{\sin^2 \frac{N\delta}{2}}{N^2 \sin^2 \frac{\delta}{2}}$$

Max

pro $\delta \to 2m\pi$: $\quad I(\vec{r}) \to I_0 N^2$

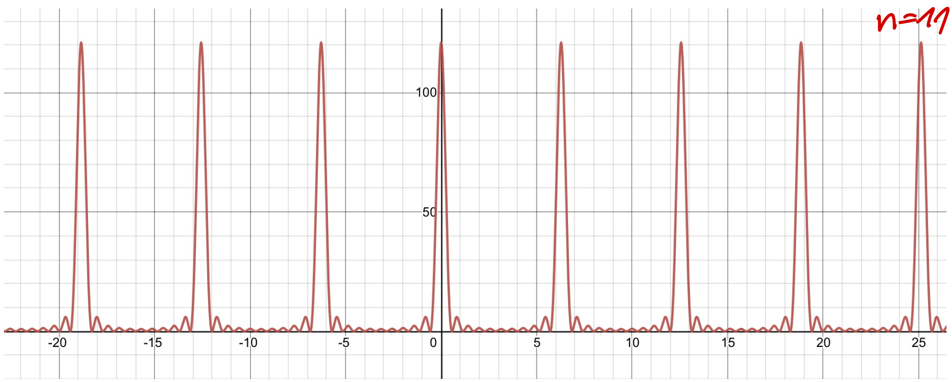
Min

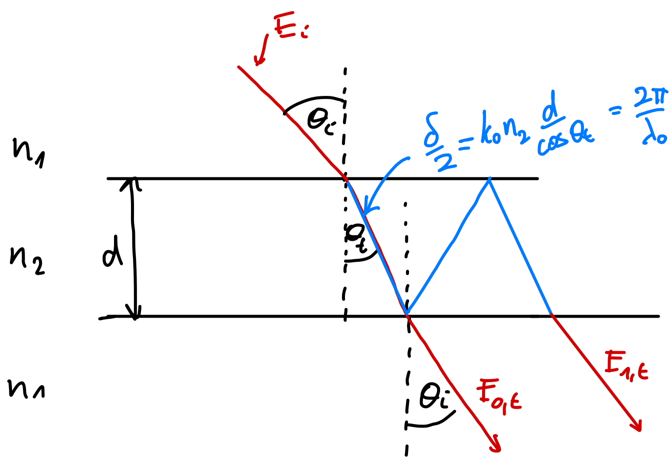
$$\sin^2 N\frac{\delta}{2} = 0 \quad \wedge \quad \sin^2 \frac{\delta}{2} \neq 0$$

$$N\frac{\delta}{2} = p\pi \qquad p \in \mathbb{Z}$$

$$\delta = 2\pi \frac{p}{N} \quad \wedge \quad \frac{\delta}{2} \neq m\pi \ \to \delta \neq 2m\pi$$

$$f(x) = \frac{\sin^2\left(\frac{nx}{2}\right)}{\sin^2\left(\frac{x}{2}\right)}$$
$n = 11$





$$\frac{\delta}{2} = k_0 n_2 \frac{d}{\cos\theta_t} = \frac{2\pi}{\lambda_0} n_2 \frac{d}{\cos\theta_t}$$

$$E_{0,t} = t_{12} t_{21} e^{i\frac{\delta}{2}} E_i$$

$$E_{1,t} = t_{12} r_{21} r_{21} t_{21} e^{i\frac{\delta}{2}} e^{i\delta} E_i$$

$$E_{2,t} = t_{12} t_{21} r_{21}^4 e^{i\frac{\delta}{2}} e^{i2\delta} E_i$$

$$\vdots$$

$$E_{l,t} = t_{12} t_{21} r_{21}^{2l} e^{i\frac{\delta}{2}} e^{i\delta l} E_i$$

$$E_{tot} = \sum_{\ell=0}^{\infty} E_{\ell,t} = t_{12}\, t_{21}\, e^{i\frac{\delta}{2}} E_i \underbrace{\sum_{\ell=0}^{\infty} r_{21}^{2\ell} e^{i\delta\ell}}_{(r_{21}^2 e^{i\delta})^\ell} = t_{12}\, t_{21}\, e^{i\frac{\delta}{2}} E_i \frac{1}{1-r_{21}^2 e^{i\delta}}$$

$$(1-r_{21}^2 e^{i\delta})(1-r_{21}^2 e^{-i\delta}) = 1 + r_{21}^4 - 2 r_{21}^2 \cos\delta$$

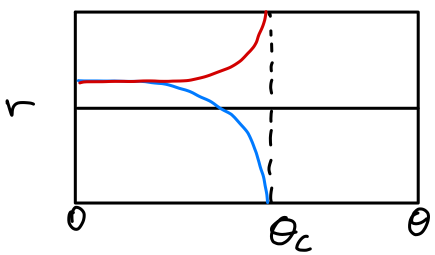
$$r_{21} = r$$

$$I = \frac{1}{4}\varepsilon_0 n_1^2 E_{tot} E_{tot}^* = \frac{1}{4}\varepsilon_0 n_1^2 \underbrace{t_{12}^2\, t_{21}^2}_{1-r_{12}^2 = 1-r_{21}^2} E_0^2 \frac{1}{1+r^4-2r^2\cos\delta} =$$

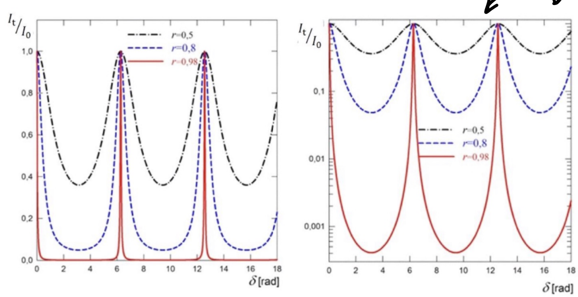
$$= \frac{1}{4}\varepsilon_0 n_1^2 E_0^2 \frac{(1-r^2)^2}{(1-r^2)^2 + 2r^2(1-\cos\delta)} =$$

$$= \frac{1}{4}\varepsilon_0 n_1^2 E_0^2 \frac{1}{1+\frac{2r^2}{(1-r^2)^2}(1-\cos\delta)} = \frac{I_0}{1+\frac{4r^2}{(1-r^2)^2}\sin^2\frac{\delta}{2}} =$$

$$I = \frac{I_0}{1+F\sin^2\frac{\delta}{2}} \qquad F = \frac{4r^2}{(1-r^2)^2} = \frac{4R}{(1-R)^2} \quad \ldots \text{jemnost}$$



log scale



Skládání vln různých frekvencí

2 vlny:

$$E_1 = E_0 \cos(\vec{k}_1\cdot\vec{r} - \omega_1 t) \qquad k_1 = \frac{\omega_1}{c} n$$

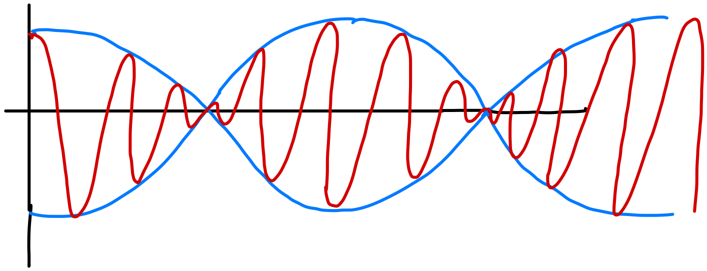
$$E_2 = E_0 \cos(\vec{k}_2\cdot\vec{r} - \omega_2 t) \qquad k_2 = \frac{\omega_2}{c} n$$

$$E = E_0 \cos\left[\underbrace{\left(\frac{k_1+k_2}{2}\right)}_{\bar{k}} z - \underbrace{\left(\frac{\omega_1+\omega_2}{2}\right)}_{\bar{\omega}} t\right] \cos\left[\underbrace{\left(\frac{k_1-k_2}{2}\right)}_{\delta k} z - \underbrace{\left(\frac{\omega_1-\omega_2}{2}\right)}_{\delta\omega} t\right]$$

$$= E_0 \cos(\bar{k} z - \bar{\omega} t)\cos(\delta k \cdot z - \delta\omega \cdot t)$$

nech $\bar{k} \gg \delta k$:

$$E = E_0 \cos(\bar{k}z - \bar{\omega}t)\cos(\delta k \cdot z - \delta\omega \cdot t)$$

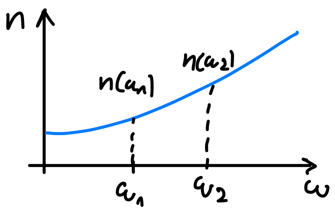


$$v_p = \frac{\bar{\omega}}{\bar{k}} = \frac{\omega_1+\omega_2}{k_1+k_2} = c$$

$$v_g = \frac{d\omega}{dk} = \frac{\omega_1-\omega_2}{k_1-k_2} = c$$

pre vákuum

1) Normální disperze

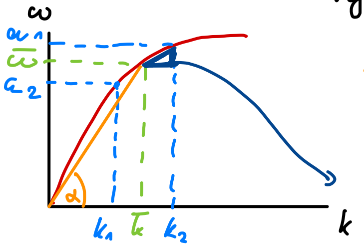


$$v_p = \frac{\bar{\omega}}{\bar{k}} = \frac{\omega_1+\omega_2}{k_1+k_2} = c\,\frac{\omega_1+\omega_2}{n_1\omega_1+n_2\omega_2}$$

$$v_g = \frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{1}{\frac{n(\omega)}{c} + \frac{\omega}{c}\frac{dn(\omega)}{d\omega}} = \frac{c}{n(\omega)+\omega\, n'(\omega)} = \frac{\frac{c}{n}}{1+\frac{\omega}{n}n'} = \frac{v_p}{1+\frac{\omega n'}{n}}$$

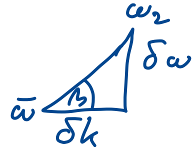
$k = \frac{\omega}{c}n(\omega)$ → Disperzní vztah: $\omega = f(k)$

pro normální disperzi: $v_g < v_p$



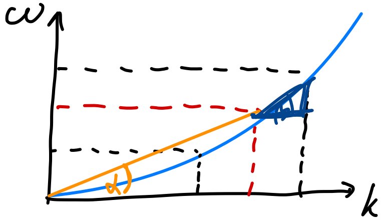
$\mathrm{tg}\,\alpha = \frac{\bar{\omega}}{\bar{k}} = v_p$

z obrázku vidět $v_p > v_g$



$\mathrm{tg}\,\beta = \frac{d\omega}{dk} = v_g$

2) Anomální disperze



$tg\,\beta > tg\,\alpha \Rightarrow v_g > v_p$

$$E = \underbrace{E_0 \cos(\delta k z - \delta\omega t)}_{E_0'} \cos(\bar{k} z - \bar{\omega} t)$$

$$w_E = \frac{1}{4}\varepsilon_0 \varepsilon_r E_0'^2 \sim \cos^2(\delta k z - \delta\omega t) \Leftrightarrow v_g = \frac{d\omega}{dk}$$ → energie se šíří grupova rychlostí

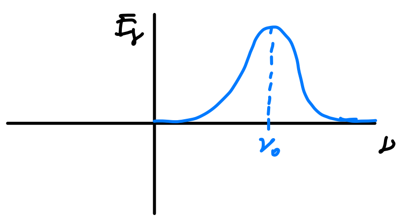
Pulz vlny

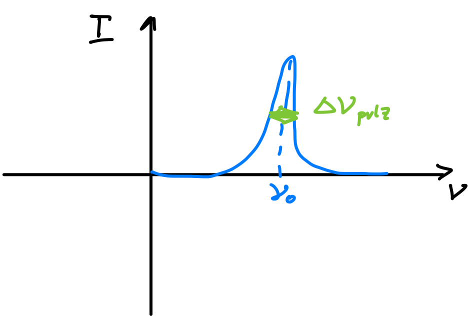
$$E(z,t) = \int_{-\infty}^{\infty} E_\nu(\nu)\, e^{i(k_0 z - \overbrace{2\pi\nu}^{\omega} t)}\, d\nu$$

amplituda $[E_\nu] = \frac{V}{mHz}$; i u sumy bylo $[E_0] = \frac{V}{m}$

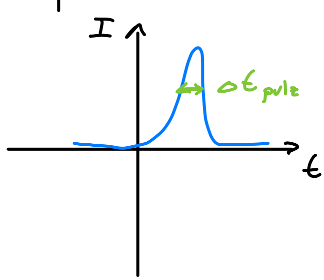
- Gausovský pulz

$$E_\nu(\nu) = \frac{E_0 a}{\sqrt{\pi}} e^{-a^2(\nu-\nu_0)^2}$$





$\longleftrightarrow$



$$\Delta\nu_{pulz} \cdot \Delta t_{pulz} = \frac{2\ln 2}{\pi}$$

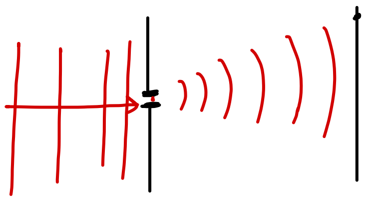
Difrakce

– skalární teorie (Fresnel, Fraunhofer)

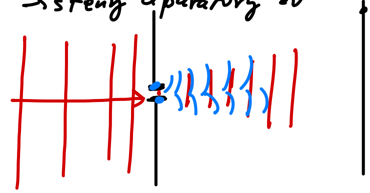
– Huyghens-Fresnelův princip

$E = E(x, y, z, t)$

$I = EE^*$



alternatívní model
→ steny aparatury sú zdroj vln

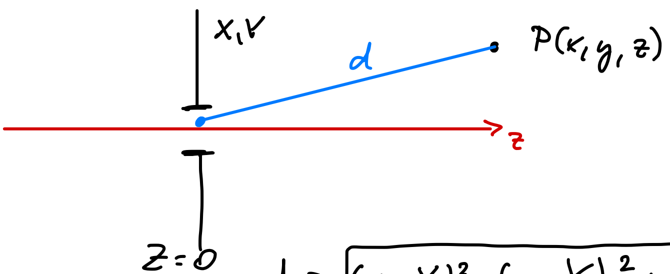


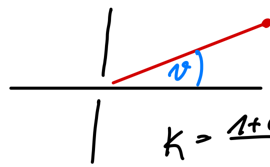
Difrakčný integrál

$$\boxed{E(P) = -\frac{i}{\lambda} \iint_{apert} E_A(X, Y) \frac{e^{ikd}}{d} K(\vartheta)\, dX\, dY}$$

$K(\vartheta)$ → uhlový faktor

→ Paraxiální aproximace $K(\vartheta) \approx 1$





$K = \frac{1 + \cos\vartheta}{2}$

$d = \sqrt{(x-X)^2 + (y-Y)^2 + z^2}$

Fresnelova aproximace

$d \to z \qquad k = \frac{2\pi}{\lambda} d \qquad$ Taylor kolem $\vartheta \approx 0$

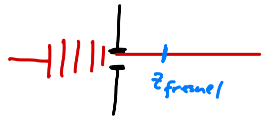
$$d = z\sqrt{1 + \frac{(x-X)^2}{z^2} + \frac{(y-Y)^2}{z^2}} \approx z\left(1 + \frac{(x-X)^2}{2z^2} + \frac{(y-Y)^2}{2z^2}\right)$$

$$d = z + \frac{(x-X)^2}{2z} + \frac{(y-Y)^2}{2z} = z + \frac{x^2+y^2}{2z} + \frac{X^2+Y^2}{2z} - \frac{xX + yY}{z}$$

$$E(P) = -\frac{i}{\lambda} E_0 \iint e^{ikz} e^{ik\left(\frac{x^2+y^2}{z}\right)} e^{ik\left(\frac{X^2+Y^2}{z^2}\right)} e^{-ik\left(\frac{xX+yY}{z}\right)} dX dY =$$

$$E(P) = -\frac{i}{\lambda} E_0 e^{ikz} e^{\frac{ik(x^2+y^2)}{2z}} \iint e^{ik\left(\frac{X^2+Y^2}{z^2}\right)} e^{-ik\left(\frac{xX+yY}{z}\right)} dX dZ$$

Fresnelova aproximace, platí pro $(x-X)^2+(y-Y)^2 \ll 2z$



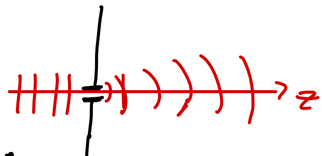
Frauenhofova aproximace $e^{ik\left(\frac{X^2+Y^2}{z^2}\right)} \approx 1$; $2z \gg k(X^2+Y^2)$

$$2z \gg \frac{2\pi}{\lambda}\left(\frac{D}{2}\right)^2 = \frac{2\pi}{\lambda}\frac{D^2}{4}$$

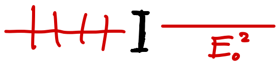
$$E(0,0,z) = E_0\left(e^{ikz} - e^{ik\sqrt{\left(\frac{D}{2}\right)^2+z^2}}\right)$$

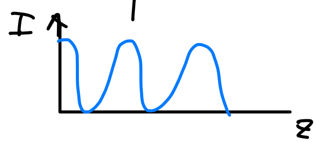
$$I = EE^* = E_0^2\, 2\left(1-\cos k\left(\sqrt{\left(\frac{D}{2}\right)^2+z^2} - z\right)\right)$$

$$I \in (0, 4E_0)$$



Dopadaní na terčík





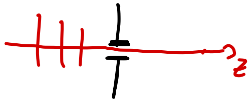
Babinetův princip

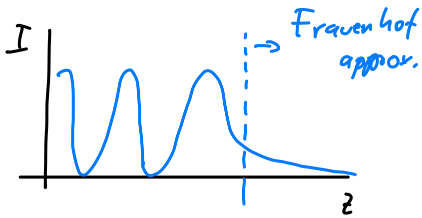
$E_A + E_{TE} = E_\Sigma$ ← rovinná vlna → sčítání vln

$$E_\Sigma = E_0 e^{ikz} = E_\emptyset + E_{\emptyset TE}$$

$$E_{\emptyset TE} = E_0 e^{ikz} - E_A = E_0 e^{ikz} - E_0 e^{ikz} + E_0 e^{ik\sqrt{\left(\frac{D}{2}\right)^2+z^2}}$$

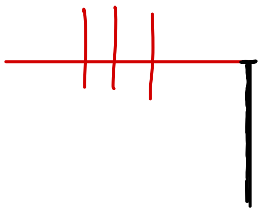
$$E_{\emptyset TE} = E_0 e^{ik\sqrt{\left(\frac{D}{2}\right)^2+z^2}} \qquad I_{\emptyset TE}(0,0,z) = E_0^2$$ → Poissonova skvrna

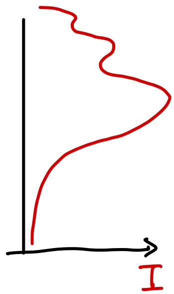




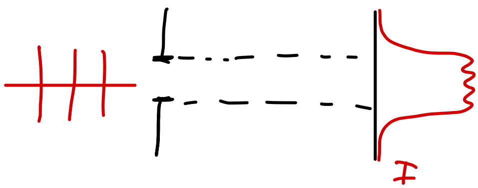
: $I(0,0,z) = \frac{1}{\lambda^2 z^2} E_0^2 \pi^2 \left(\frac{D}{2}\right)^2$

Polrovinová prekážka

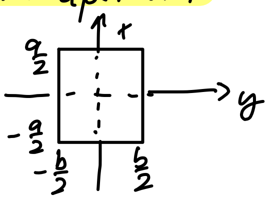




Štrbina



Obdelníková apertúra



$$E(P) = -\frac{i}{\lambda} E_0 \frac{1}{z} e^{ikz} e^{ik\left(\frac{x^2+y^2}{2z}\right)} \int e^{-ik\left(X\frac{x}{z}\right)} dX \int e^{-ik\left(Y\frac{y}{z}\right)} dY$$

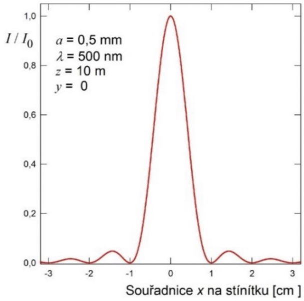
$$\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-ik\left(X\frac{x}{z}\right)} dX = -\frac{z}{ikx}\left(e^{-\frac{ikax}{2z}} - e^{+\frac{ikaz}{2z}}\right) = \frac{2z}{kx}\sin\left(\overset{u_{pa}}{\frac{akx}{2z}}\right) = a\,\frac{\sin\frac{akx}{2z}}{\frac{akx}{2z}}$$

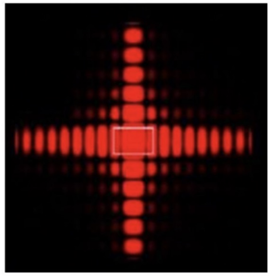
→ rovnako aj pre $\int dY$

$$I = E(P)E^*(P) = \underbrace{\frac{E_0^2}{\lambda^2 z^2} a^2 b^2}_{I_0} \frac{\sin^2 u_{pa}}{u_{pa}^2} \frac{\sin^2 v_{pa}}{v_{pa}^2}$$

$$u_{pa} = \frac{akx}{2z}$$

$$v_{pa} = \frac{bky}{2z}$$



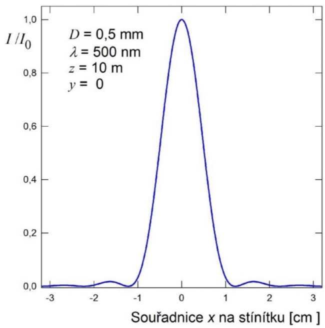


→ nulové body

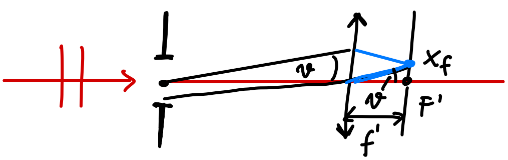
$m\frac{\lambda z}{a}$ $\quad m\in\mathbb{Z}$

Kruhová aparatura

→ Besselovy funkce



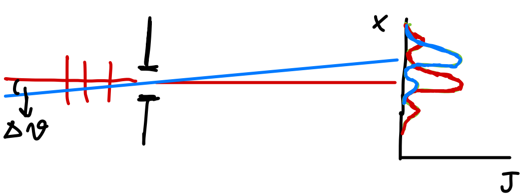
Zobrazení Fraunhoferova dif. obrazce



$P(x, y, z)$

$$\vartheta = \frac{x}{z} = \frac{x_z}{z}$$

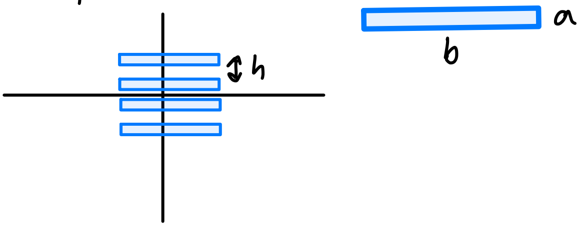
Rayleigho kritérium rozlišitelnosti 2 bodů

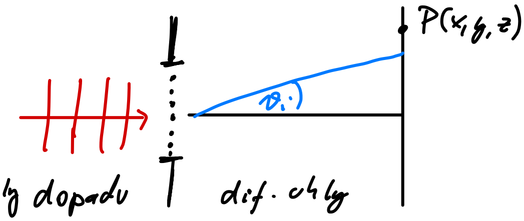


→ první maximum posunutého bodu je v prvním minimu středového bodu

$$\Delta\vartheta_{min} = \frac{x}{z} = 1{,}22\,\frac{\lambda}{D}$$

Difrakce na mřížce





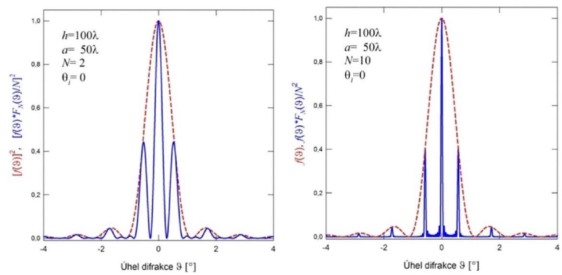
úhly dopadu
$\theta_i \dots x$
$\varphi_i \dots y$

dif. úhly
$\vartheta \dots x$
$\varphi \dots y$

$\Big\}$ velké pro dif mřížku

$$I = I_0 \left(\frac{\sin^2 N \frac{\delta}{2}}{N^2 \sin^2 \frac{\delta}{2}} \right) \frac{\sin^2 u}{u^2} \frac{\sin^2 v}{v^2}$$

$$u = \frac{hk \sin\vartheta}{2} \qquad v = \frac{hk \sin\varphi}{2}$$



→ maxima:

$$\frac{hk \sin\vartheta}{2} = m\pi$$

$$h \sin\vartheta_{max} = m\lambda$$

$$h(\sin\vartheta_{max} - \sin\theta_i) = m\lambda$$

Úhlové rozložení $\quad h\cos\vartheta \, d\vartheta = m \, d\lambda$

$$D_\vartheta = \frac{d\vartheta}{d\lambda}\bigg|_{\theta_i = konst} = \frac{m}{h\cos\vartheta} \qquad \Delta\vartheta = D_\vartheta \Delta\lambda$$

Volný spektrální interval $\qquad$ spektrum (λ_1, λ_2) $\quad \lambda_2 > \lambda_1$

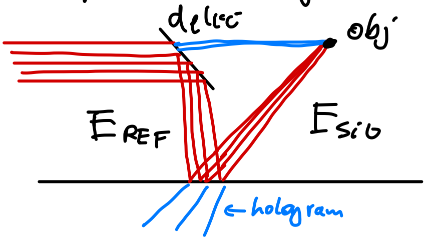
$$I = \lambda_2 - \lambda_1 \qquad m\lambda_2 = (m+1)\lambda_1$$

$$h \sin\vartheta_{sm} = (m+1)\lambda_1 = m\lambda_2 \qquad \lambda_2 - \lambda_1 = \left(\frac{m+1}{m}\right)\lambda_1 - \lambda_1 = \frac{\lambda_1}{m}$$

Holografie

Záznam fáze

IF referenční a signální vlny



$$\tilde{E}_{REF} = A_{REF}\, e^{i(\vec{k}_{REF}\cdot\vec{r} - \omega t)} = A_{REF}\, e^{i(k_{REF}x - \omega t)} = A_{REF}\, e^{i(k_{REF}\sin\alpha\, x - \omega t)}$$

$$E_{sig} = A_{sig}\, e^{i(k_{sig}\sin\beta\, x - \omega t)}$$

$$k_{REF}\sin\alpha\, x = -\varphi(x)$$

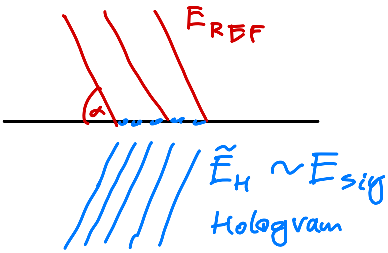
$$k_{sig}\sin\beta\, x = -\theta(x)$$

$$I \sim (\tilde{E}_{REF} + \tilde{E}_{sig})(E_{REF} + E_{sig})^{*} = A_{REF}^2 + A_{sig}^2 + 2A_{REF}A_{sig}\cos(\varphi - \theta)$$

amplituda propust.

$$t_F = \underbrace{t_0}_{\text{původní}} - \underbrace{a}_{\text{koef. sčernění}}\underbrace{\left(A_{REF}^2 + A_{sig}^2 + 2A_{REF}A_{sig}\cos(\varphi - \theta)\right)}_{\text{záznam}}$$

Čtení záznamu



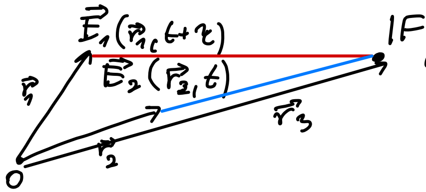
$$E_H = t_F E_{REF} = t_0 \vec{E}_{REF} - a(A_{REF}^2 - A_{sig}^2)E_{REF} - a A_{REF}^2 A_{sig} e^{-i(2\varphi - \theta)} - a A_{REF}^2 \underbrace{A_{sig} e^{-i\theta}}_{E_{sig}}$$

Koherence

Kvazi-monochromatické záření

$$\Delta\omega\,\Delta t = \text{konst.}$$

Korelace 2 el. polí



detektor, t_d - integrační doba

Korelační funkce $\Gamma_{12}(\vec r_1, \vec r_2, \tau) = \langle E_1(\vec r_1, t+\tau) E_2^*(\vec r_2, t)\rangle = \frac{1}{t_d}\int_{-\frac{t_d}{2}}^{\frac{t_d}{2}} E_1 E_2^* \, dt$

$$I_d(\vec r_3) = \frac{1}{4}\varepsilon_0\varepsilon_r \langle(\vec E_1 + \vec E_2)(\vec E_1 + \vec E_2)^*\rangle_{t_d} = \frac{1}{4}\varepsilon_0\varepsilon_r \overbrace{\langle \vec E_1 \vec E_1^* \rangle_{t_d}}^{\Gamma_{11}(\vec r_1, \vec r_1, 0)} + \frac{1}{4}\varepsilon_0\varepsilon_r \overbrace{\langle \vec E_2 \vec E_2^* \rangle}^{\Gamma_{22}(\vec r_2, \vec r_2, 0)}$$
$$+ \frac{1}{4}\varepsilon_0\varepsilon_r \Big(\underbrace{\langle \vec E_1 \vec E_2^* \rangle_{t_d}}_{\Gamma_{12}} + \underbrace{\langle E_1^* E_2 \rangle_{t_d}}_{\Gamma_{12}^*}\Big)$$

$$I_d(\vec r_3) = \underbrace{\frac{1}{4}\varepsilon_0\varepsilon_r \Gamma_{11}(0)}_{I_1} + \underbrace{\frac{1}{4}\varepsilon_0\varepsilon_r \Gamma_{22}(0)}_{I_2} + \frac{1}{4}\varepsilon_0\varepsilon_r 2\,\mathrm{Re}\{\Gamma_{12}(\tau)\}$$

$$\Gamma_{11}(0) = \frac{4 I_1}{\varepsilon_0\varepsilon_r}$$

$$\Gamma_{22}(0) = \frac{4 I_2}{\varepsilon_0\varepsilon_r}$$

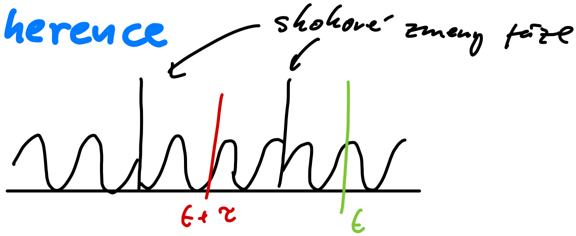
Komplexní stupeň koherence

$$\gamma_{12}(\vec r_1, \vec r_2, \tau) = \frac{\Gamma_{12}(\vec r_1, \vec r_2, \tau)}{\sqrt{\Gamma_{11}(0)\,\Gamma_{22}(0)}}$$

$$I_d(\vec r_3) = I_1 + I_2 + \frac{1}{2}\varepsilon_0\varepsilon_r \sqrt{\Gamma_{11}\Gamma_{22}}\,\mathrm{Re}\{\gamma_{12}\}$$

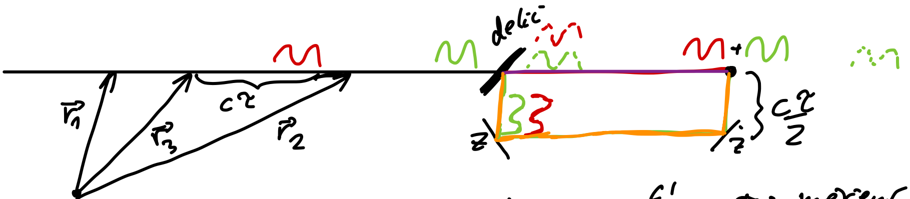
$$\boxed{I_d(\vec r_3) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}\{\gamma_{12}(\vec r_1, \vec r_2, \tau)\}}$$

Časová koherence



$$\Gamma_{12}(\vec{r}_3,\vec{r}_3,\tau)=\Gamma_{12}(\vec{r}_1,\vec{r}_2,0)$$

τ podélná koherence



$$E_1(t')=E_0\,e^{i(\varphi(t+\tau)-\omega_0(t+\tau))}$$
$$E_2(t')=E_0\,e^{i(\varphi(t)-\omega_0 t)}$$

t' ... čas měření

φ ... skoky fáze

$\varphi(t+\tau)=\varphi_1$

$\varphi(t)=\varphi_2$

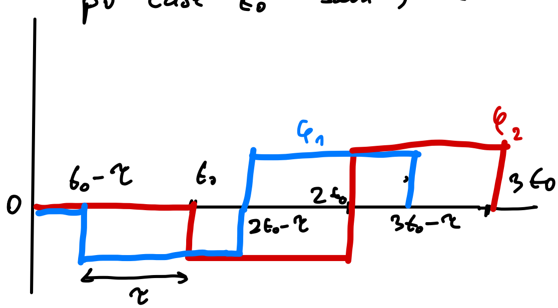
$$I_d=I_1+I_2+2I_1I_2\,\mathrm{Re}\{\gamma_{12}(\tau)\}$$

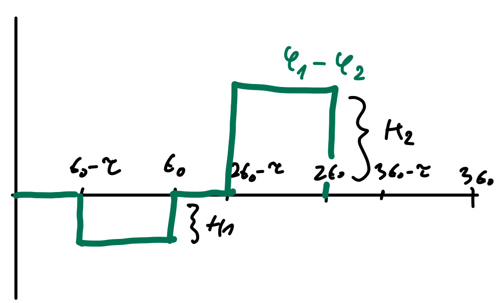
$$\gamma(\tau)=\frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\,\Gamma_{22}(0)}}$$

$$\Gamma_{11}(0)=\langle E_1E_1^*\rangle=\frac{1}{t_d}\int_{-\frac{t_d}{2}}^{\frac{t_d}{2}}E_0^2\,dt=E_0^2$$

$$\Gamma_{22}(0)=\langle E_2E_2^*\rangle=E_0^2$$

po čase t_0 – skok fáze





$$\Gamma_{12}(\tau) = \langle E_1(t+\tau) E_2^*(t) \rangle_{td} = \langle E_0 e^{-i\omega_0 \tau} e^{i(\varphi_1-\varphi_2)} \rangle_{td}$$

$$= \frac{E_0^2 e^{-i\omega_0\tau}}{N t_0}\left(\int_0^{t_0-\tau} e^{i0}\,dt + \int_{t_0-\tau}^{t_0} e^{iH_1}\,dt + \int_{t_0}^{2t_0-\tau} e^{i0} + \int_{2t_0-\tau}^{2t_0} e^{iH_2}\,dt + \dots \right)$$

$$= E_0^2 e^{-i\omega_0\tau} \frac{1}{N t_0}\left(N(t_0-\tau) + \tau \underbrace{\sum_{n=1}^{N} e^{iH_n}}_{=0} \right)$$

$H_1, \dots, H_n$ náhodná

$e^{iH_n} = \cos H_n + i \sin H_n$

$\Rightarrow \langle e^{iH_n} \rangle = 0$

$$= E_0^2 e^{-i\omega_0\tau} \frac{1}{N t_0}\left(N(t_0-\tau) \right) =$$

$$\Gamma_{12}(\tau) = \left(1 - \frac{\tau}{t_0}\right) E_0^2 e^{-i\omega_0\tau}$$

$$\gamma_{12}(\tau) = \frac{E_0^2 e^{-i\omega_0\tau}\left(1-\frac{\tau}{t_0}\right)}{E_0 \cdot E_0} = \underbrace{e^{-i\omega_0\tau}\left(1-\frac{\tau}{t_0}\right)}_{0 \le \tau \le t_0} = \underbrace{e^{-i\omega_0\tau}\left(1-\frac{|\tau|}{t_0}\right)}_{\tau<0 \text{ aj } \tau>0}$$

$$\mathrm{Re}\{\gamma_{12}(\tau)\} = \cos \omega_0\tau \left(1-\frac{\tau}{t_0}\right)$$

$$I = 2I_0 + 2I_0 \,\mathrm{Re}\{\gamma_{12}(\tau)\} = 2I_0 + 2I_0\left(1-\frac{|\tau|}{t_0}\right)\cos\omega_0\tau$$

$$I = 2I_0\left(1 + \left(1-\frac{|\tau|}{t_0}\right)\cos\omega_0 t\,\tau\right)$$

$$I_{max} = 2I_0\left(1+\left(1-\frac{|\tau|}{t_0}\right)\right) = 2I_0\left(2-\frac{|\tau|}{t_0}\right)$$

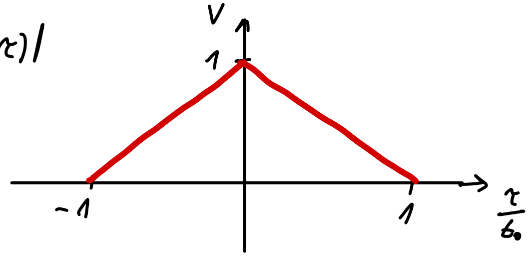
$$I_{MIN} = 2I_0\left(1-\left(1-\frac{|\tau|}{t_0}\right)\right) = 2I_0\frac{|\tau|}{t_0}$$

$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \left(1-\frac{|\tau|}{t_0}\right) = |\tilde{\gamma}_{12}(\tau)|$$

↓

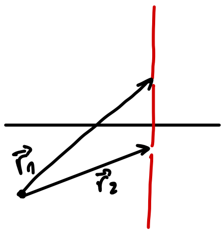
viditelnosť

(meraní)

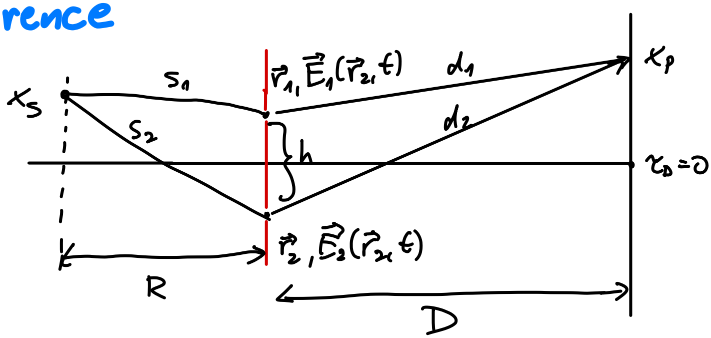


Pro $\tau > t_0$: $I_d = I_1 + I_2$

Příčná koherence



Youngov pokus

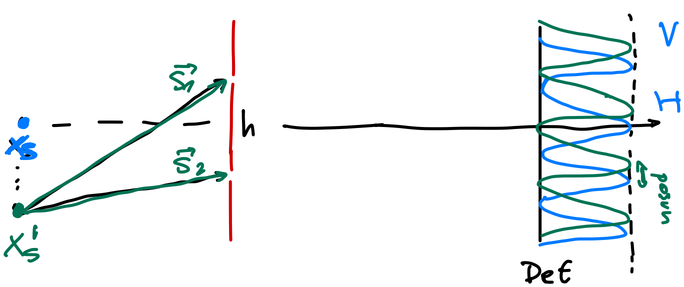


$s_2 - s_1 \approx \frac{h x_s}{R}$ ... dráhový rozdiel ... $d_2 - d_1 \approx \frac{h x_p}{D}$

$\frac{k h x_s}{R}$ ... fázový rozdiel ... $\frac{k h x_p}{D}$

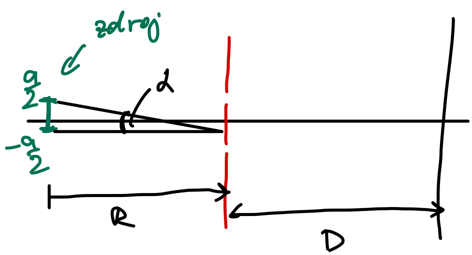
$s_2 - s_1 = c\tau_R$ $\qquad$ $d_2 - d_1 = c\tau_D$

$$\tau = \tau_R + \tau_s$$

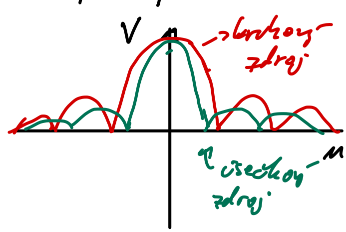


Zdroj – zcela koherentní $t_0 \gg \tau$

2 nezávislé zdroje (navzájom nezávislé) $\Rightarrow I = I_1 + I_2$, $V = \left|\cos\frac{kah}{2R}\right|$



$$V = \frac{\sin\frac{kah}{2R}}{\frac{kah}{2R}}$$



$$u = \frac{kah}{2R}$$

$\alpha = \frac{a}{R}$ → uhlový rozmer zdroja

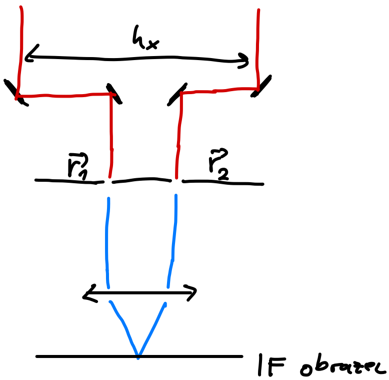
Jak zvýšit koherenci?

1) zvýšit R

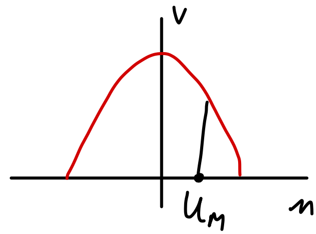
2) zmenšit zdroj (a)

3) zmenšit k

Michelsonův stelární interferometr



$$V = \frac{\sin \frac{k a h_x}{2R}}{\frac{k a h_x}{2R}}$$



$$u_M = \frac{k a h_x}{2R} \qquad \frac{a}{R} = \frac{2\pi}{k h_x}$$

Geometrická optika

- šíření svět. pole v nehomogenním prostředí
- interakce s objekty $\gg \lambda$

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} \pm \omega t \pm \delta)}$$

$$\vec{k}\cdot\vec{r} = k_0 \underbrace{n\,\vec{s}_0\cdot\vec{r}}_{\text{optická dráha}}$$

Nehomog. prostředí $\rightarrow$ $\varepsilon_r = \varepsilon_r(\vec{r}) \Rightarrow n = n(\vec{r})$

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(k_0\varphi(\vec{r}) - \omega t)}$$
$$\vec{B}(\vec{r},t) = \vec{B}_0 e^{i(k_0\varphi(\vec{r}) - \omega t)}$$

$\varphi(\vec{r})$... eikonál

$$\nabla\times\left(\vec{E}_0 e^{i(k_0\varphi - \omega t)}\right) = \cancel{e^{i(k_0\varphi-\omega t)}}\,\nabla\times\vec{E}_0 + ik_0\nabla\varphi\times\vec{E}_0\,\cancel{e^{i(\,)}} = i\omega\mu_0\vec{H}_0(\vec{r})\cancel{e^{i(\,)}}$$

$$\nabla\times\vec{E}_0 + ik_0\nabla\varphi\times\vec{E}_0 = i\omega\mu_0\vec{H}_0$$

$$\underbrace{\frac{1}{ik_0}\nabla\times\vec{E}_0}_{\sim\lambda_0\nabla\times\vec{E}_0} + \nabla\varphi\times\vec{E}_0 = \frac{\omega}{k_0}\mu_0\vec{H}_0$$

pro $\lambda_0 \ll$ prost. $\vec{E}_0(\vec{r})$ můžeme zanedbat

$$\nabla\varphi\times\vec{E}_0 = \frac{\omega}{k_0}\mu_0\vec{H}_0 \rightarrow \vec{H}_0 = \frac{k_0}{\omega\mu_0}\nabla\varphi\times\vec{E}_0$$

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \xrightarrow{\text{analogicky}} \vec{E}_0 = -\frac{k_0}{\varepsilon_0 n^2\omega}\nabla\varphi\times\vec{H}_0$$

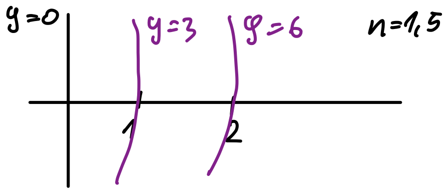
$$\nabla\cdot\vec{B} = 0 \rightarrow \mu_0\nabla\cdot\vec{H} = 0 \rightarrow \nabla\cdot(\vec{H}_0 e^{i(\,)}) = 0 \rightarrow \nabla\varphi\cdot\vec{H}_0 = 0$$
$$\nabla\cdot\vec{D} = 0 \rightarrow \nabla\cdot\vec{E} = 0 \rightarrow \nabla\cdot(\vec{E}_0 e^{i(\,)}) = 0 \rightarrow \nabla\varphi\cdot\vec{E}_0 = 0$$

$$\vec{E}_0 = -\frac{k_0^2}{\varepsilon_0 n^2 \omega^2 \mu_0}\left(\overset{a}{\nabla}\varphi \times (\overset{b}{\nabla}\varphi \times \overset{c}{\vec{E}_0})\right) = -\frac{1}{n^2}\left(\nabla\varphi(\underbrace{\cancel{\nabla\varphi\cdot\vec{E}_0}}_{0}) - \vec{E}_0(\nabla\varphi\cdot\nabla\varphi)\right)$$

$$\vec{E}_0 = \vec{E}_0 \frac{(\nabla\varphi)^2}{n^2} \quad\Rightarrow\quad \boxed{\nabla\varphi(\vec{r}) = n(\vec{r})\,\underbrace{\vec{S}_0(\vec{r})}_{\text{smer } \nabla\varphi}} \rightarrow \text{. rovnice eikonálu}$$

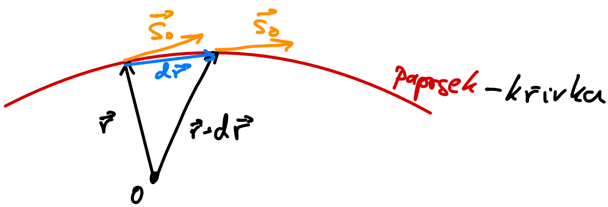
Homog. prost. $n = \text{konst}$

$$\frac{\partial\varphi}{\partial z} = n \;\rightarrow\; \varphi = nz + \varphi(z=0)$$



Paprsková rovnice

Paprsek je křivka, k níž je $\vec{s}_0(\vec{r})$ tečný



• parametrický popis $\vec{r} = \vec{r}(s)$ → skalár

$$\vec{s}_0(\vec{r}) = \frac{\nabla\varphi}{|\nabla\varphi|} = \frac{d\vec{r}}{|d\vec{r}|} = \left.\frac{d\vec{r}}{ds}\right|_{\vec{r}} \rightarrow \text{paprsková rovnice}$$

$$\frac{d}{ds}(\nabla\cdot\varphi) = \frac{d}{ds}\left(n(\vec{r})\,\vec{s}_0(\vec{r})\right) = \nabla n(\vec{r})$$

→ pro homog. pros.: $n(\vec{r}) = \text{konst}$ $\quad n\left.\frac{d^2\vec{s}_0}{ds^2}\right|_{\vec{r}} = 0 \rightarrow \vec{r} = \vec{a} + \vec{b}s$

↳ přímka

Lagrangeov invariant

$$\nabla \varphi = n(\vec r)\,\vec s_0(\vec r)$$

$$\nabla\times\nabla\varphi = 0 = \nabla\times(n\vec s_0)$$

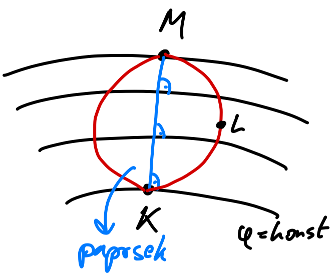
$$\int_A \nabla\times(n\vec s_0)\,dA = 0 \rightarrow \boxed{\oint_l n(\vec r)\,\vec s_0(\vec r)\cdot d\vec l = 0} \rightarrow \text{Lagrangeov invariant}$$

Fermatov princíp

– svetlo sa šíri od bodu K k bodu M po najkratšej optickej dráhe

$$|n(\vec r)\,\vec s_0(\vec r)\cdot d\vec l\,| = n(\vec r)\,|\vec s_0(\vec r)|\,dl\cos\alpha$$

→ nejkratší dráha mezi K a M je pro $\vec s_0 \parallel d\vec l$



$\vec E_0, \vec H_0 \perp \nabla\varphi$ energie $S = \vec E_0\times\vec H_0$ energie

$$\vec E_0 \sim \nabla\varphi\times\vec H_0$$
$$\vec H_0 \sim \nabla\varphi\times\vec E_0$$

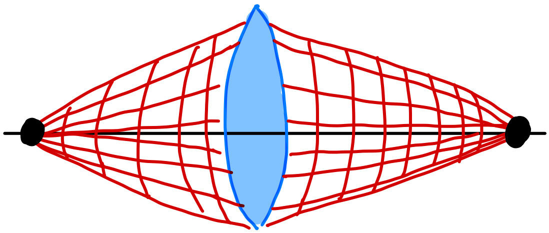
$$S\sim(\nabla\varphi\times\vec H_0)\times(\nabla\varphi\times\vec E_0) \rightarrow S\sim\nabla\varphi\underbrace{\left[(\nabla\varphi\times\vec H_0)\cdot\vec E_0\right]}_{\text{číslo}} - \vec E_0\underbrace{\left[(\nabla\varphi\times\vec H_0)\nabla\varphi\right]}_{0}$$

S má smer $\nabla\varphi$

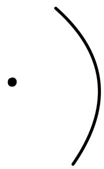
$$\delta\int_P^Q n(\vec r)\,\vec s_0(\vec r)\cdot d\vec l = 0$$

Minimum – regulární oblast = nekříží se paprsky

Stacionární bod:
zobrazování – všechny optické dráhy sú stejně dlouhé

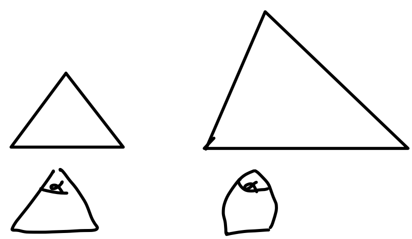


Maximum



Zobrazení

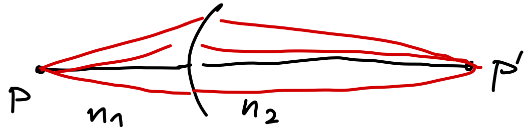
- kolineární (přímky zobrazí na přímky)
- konformní (zachovává úhly



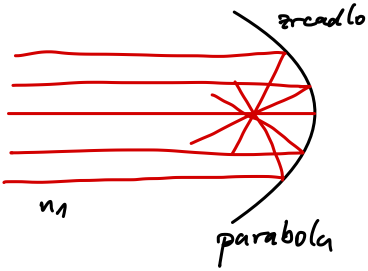
- ideální zobrazení = kolineární + konformní (absolutní optický přístroj)
≡ odraz rovinného zrcadla

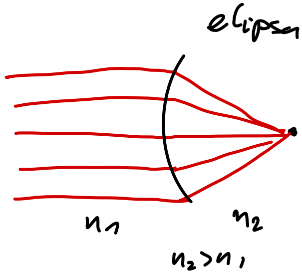
bod → bod – konjugované body
(predmet) (obraz)

René Descartes



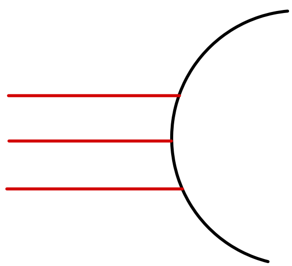
↑ plocha, kt. rozděluje rozhraní je plocha 4. řádu





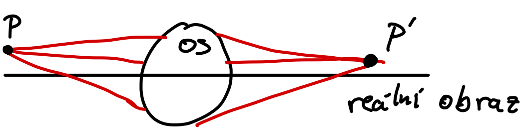
$n_1 > n_2$ .. hyperbola

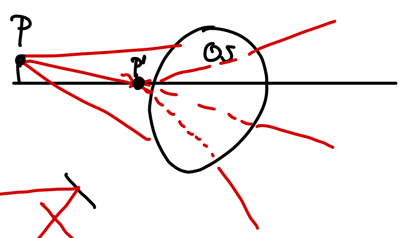
koule → aproximace Descartovy plochy



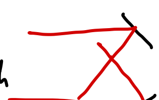
Zobrazovací soustavy

Dioptrické soustavy – paprsky vstupující na jedné straně vystupují na druhé straně

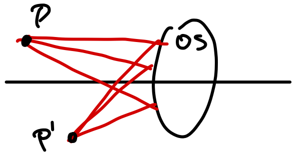


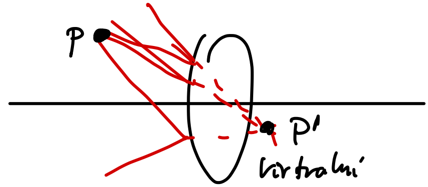


- Lomené plochy
- Sudý počet odrazových ploch



Katoptické

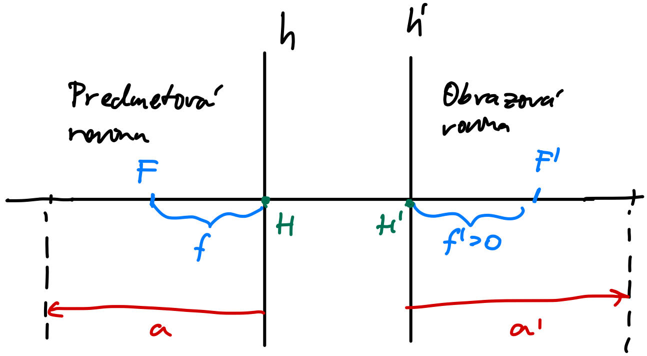




Kardinálne body optickej sústavy

1) Hlavní body
- H predmet
- H' obraz

2) Ohniskové body
- F predmet
- F' obraz



F je bod na optické ose jehož obraz leží v ∞

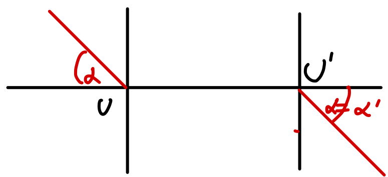
F' ——— || ——— jehož obraz predmetu leží v $-\infty$

3) Uzlové body

→ uhlové zväčšení

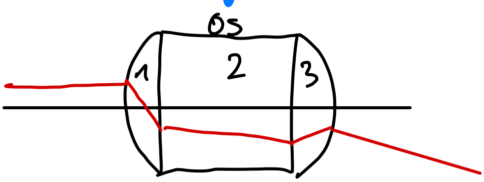
konjugovaný pŕímok je 1

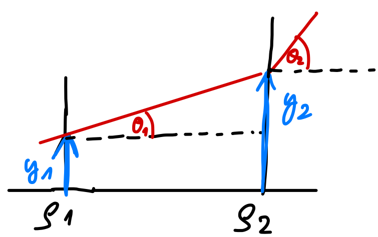
$\alpha = \alpha'$



Gaussov systém → zobrazovací rovnice

Maticová optika





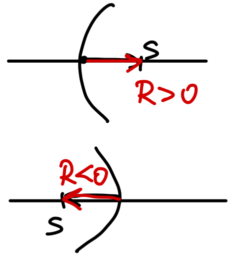
$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{\text{prenosová matice } T} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

$$\begin{pmatrix} y_n \\ \theta_n \end{pmatrix} = \overbrace{T_n T_{n-1} \dots T_2 T_1}^{OS} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

• znaménková konvence

$d>0$ $\longrightarrow$ $\qquad \uparrow\ y>0$ $\qquad \theta>0$

$d<0$ $\longleftarrow$ $\qquad \downarrow\ y<0$ $\qquad \theta<0$



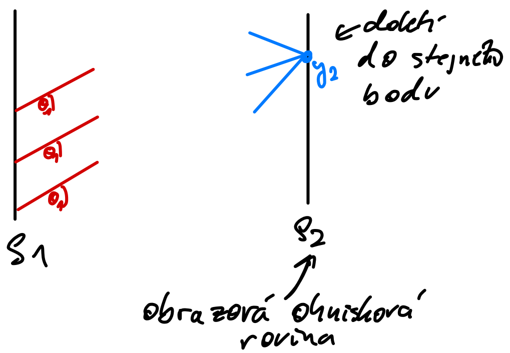
$$y_2 = y_1 + \Delta \qquad tg\,\theta_1 \approx \theta_2 = -\frac{\Delta}{d} \rightarrow \Delta = -d\theta_1$$

$$y_2 = y_1 - d\theta_1 \Rightarrow \begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & -d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

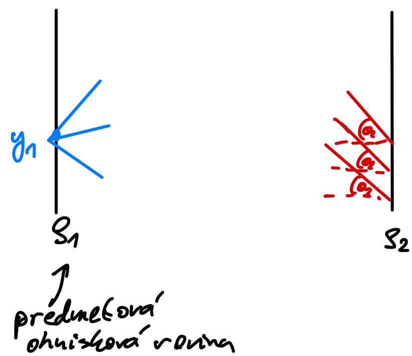
šíření mezi rovinami

$$y_2 = Ay_1 + B\theta_1$$
$$\theta_2 = Cy_1 + D\theta_1$$

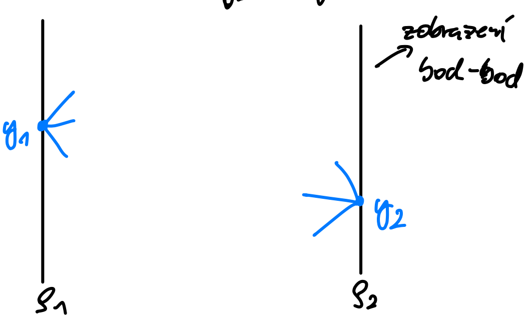
$A=0 \rightarrow y_2 = B\theta_1$



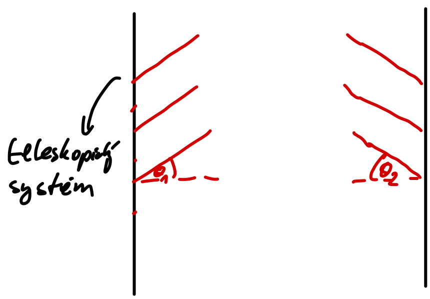
$D=0 \rightarrow \theta_2 = Cy_1$



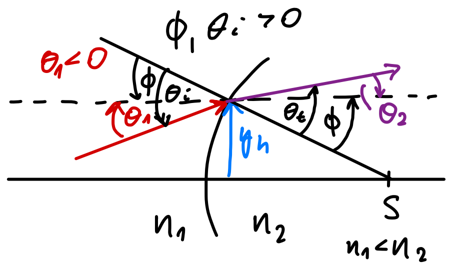
$B=0 \qquad y_2 = Ay_1$



$C=0 \qquad \theta_2 = D\theta_1$



Lom na kulovém rozhraní



$$y_h = y_1 = y_2$$

$$\phi = \overset{>0}{\theta_i} + \overset{<0}{\theta_1}$$

$$\phi = \theta_t + \theta_2$$

$$\phi \doteq \frac{y_h}{R}$$

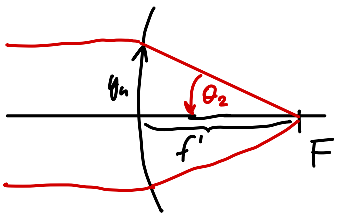
$$n_1 \theta_i = n_2 \theta_t$$

$$\theta_2 = \phi - \theta_t = \phi - \frac{n_1}{n_2}\theta_i = \phi - \frac{n_1}{n_2}(\phi - \theta_1) = \left(1 - \frac{n_1}{n_2}\right)\phi + \frac{n_1}{n_2}\theta_1$$

$$\theta_2 = \left(1 - \frac{n_1}{n_2}\right)\frac{y_1}{R} + \frac{n_1}{n_2}\theta_1$$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

← prenosová matice lomu

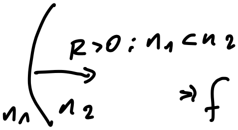


$$\theta_2 = \frac{y_1}{f'} \qquad \theta_2 = \left(1 - \frac{n_1}{n_2}\right)\frac{y_1}{R} + \frac{n_1}{n_2}\theta_1 \quad (1)$$

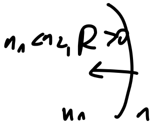
pro $\theta_1 = 0$:

$$\frac{\cancel{y_1}}{f'} = \theta_2 = \left(1 - \frac{n_1}{n_2}\right)\frac{\cancel{y_h}}{R}$$

$$\boxed{\frac{1}{f'} = \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R}} \longrightarrow \text{obrazové ohnisko}$$



$\Rightarrow f' > 0$

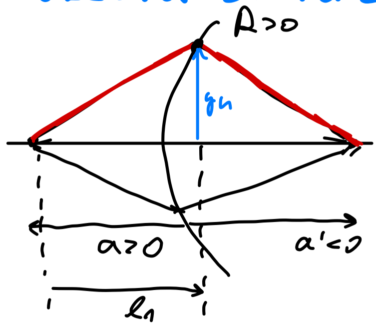


$\Rightarrow f' < 0$

předmětové ohnisko ; $Q_2 = 0$ v (1)

$$f = -\frac{n_1}{n_2} f'$$

Gaussova zobrazovací rovnice



$$T_{zob} = \begin{pmatrix} 1 & -l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f'} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} 1 & -l_1 \\ 0 & 1 \end{pmatrix} \Big\} \; B = -l_1 - l_2 \frac{n_1}{n_2} + \frac{l_1 l_2}{f'} = 0$$

$l_1 = a \qquad l_2 = a'$

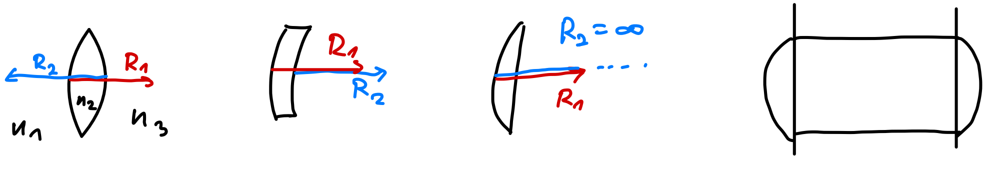
$$a - a' \frac{n_1}{n_2} - \frac{a a'}{f'} = 0 \rightarrow a'\left(\frac{a}{f'} + \frac{n_1}{n_2}\right) = a \xrightarrow[\frac{n_1}{n_2} = -\frac{f}{f'}]{f = -\frac{n_1}{n_2} f'} a'\left(\frac{a}{f'} - \frac{f}{f'}\right) = a$$

$$\rightarrow a'(a - f) = a f' \rightarrow \frac{a-f}{a} = \frac{f'}{a'} \rightarrow 1 - \frac{f}{a} = \frac{f'}{a'}$$

$$\frac{f}{a} + \frac{f'}{a'} = 1$$

Gaussova zob. rovnice

Čočka



Tenké čočky

$$T_{\text{čočka}} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_2'} & \frac{n_2}{n_3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f_1'} & \frac{n_1}{n_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f_2'} + \frac{n_2}{n_3}\frac{1}{f_1'} & \frac{n_1}{n_3} \end{pmatrix}$$

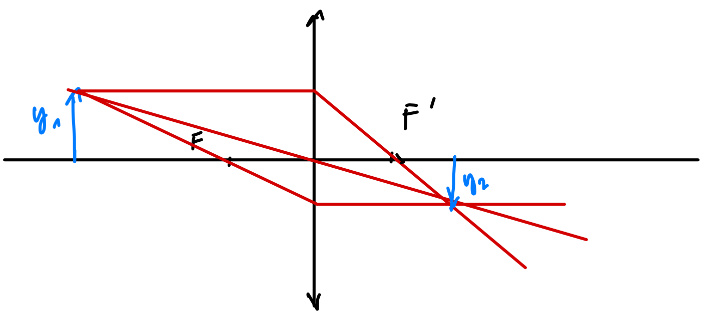
$$\frac{1}{f_1'} = \left(1 + \frac{n_1}{n_2}\right) \frac{1}{R_1}$$

$$\frac{1}{f_2'} = \left(1 - \frac{n_2}{n_3}\right) \frac{1}{R_2}$$

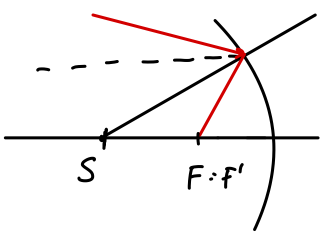
čočková rovnice: $\frac{1}{f'} = \frac{1}{f_2'} + \frac{n_2}{n_3}\frac{1}{f_1'}$

pro $n_1 = n_3 = 1 \rightarrow$ čočka ve vzduchu:

$$\frac{1}{f'} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)(n_2 - 1) = -\frac{1}{f}$$

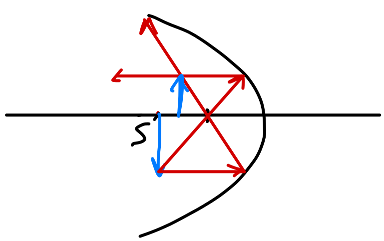


Odraz na zrcadle



$\Rightarrow$ Gaussova rovnice pro zrcadla

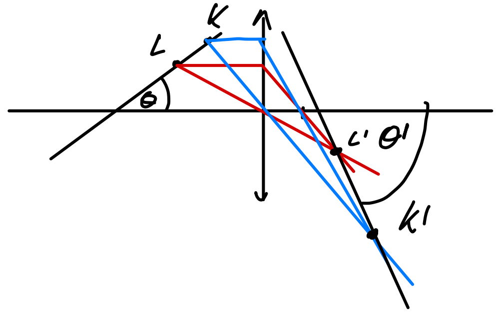
$$\frac{1}{a} + \frac{1}{a'} = \frac{2}{R} \qquad f = f' = \frac{R}{2}$$



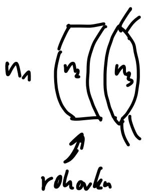
Zvětšení

1) Příčné zvětšení $M_T = \frac{y'}{y}$

2) Úhlové zvětšení $M_\theta = \frac{\theta'}{\theta} \sim \frac{tg\,\theta'}{tg\,\theta}$



Oko 2 čočka - rohovka + oční čočka

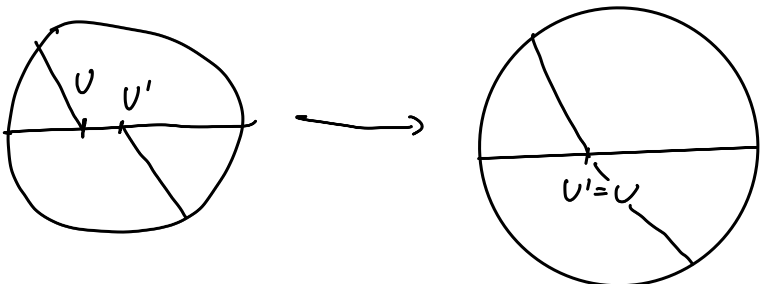


$\frac{1}{f_{oko}}$ = optická mohutnost [dioptrie]
[D]

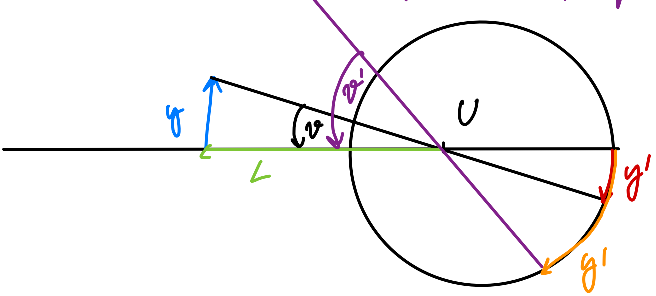
Blízká akomodace $\frac{1}{f} = 617\ D$

Vzdálená akomodace $\frac{1}{f} = 57\ D$

→ zjednodušení $U = U'$

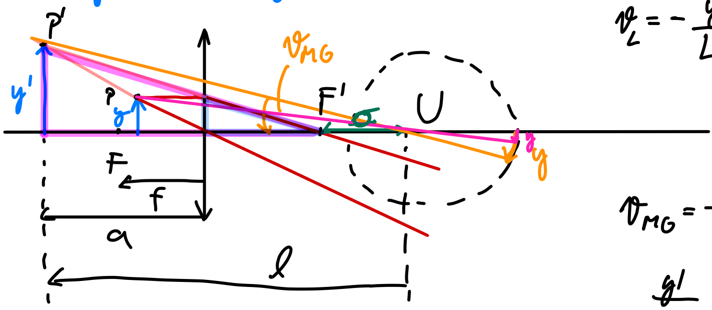


← pointa op. prístrojov = zvačšenie uhlu



Optické přístroje

Lupa



$\vartheta_L = -\frac{y}{L}$ → L = konvenční vzdálenost

$L \overset{def}{=} 25\,cm$

$\vartheta_{MG} = -\frac{y'}{l} > 0$

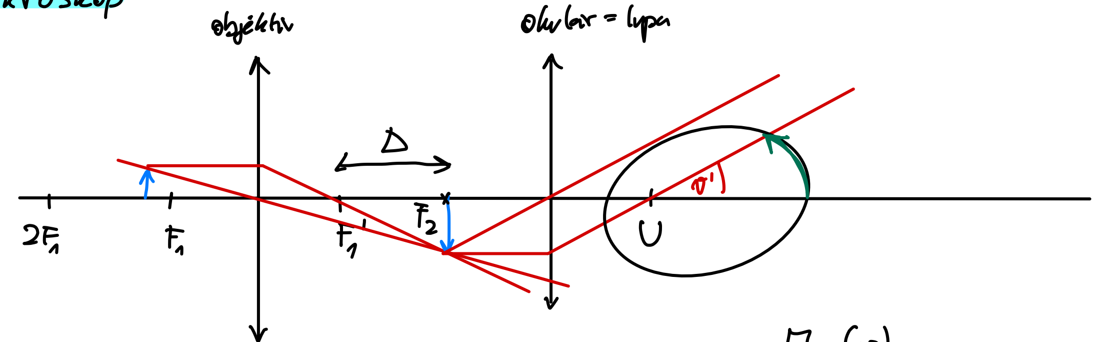
$\frac{y'}{y} = -\frac{l+\sigma}{f'}$

zvětšení lupy

$$\Gamma_{MG} = \frac{\vartheta_{MG}}{\vartheta_L} = \frac{-\frac{y'}{l}}{-\frac{y}{L}} = \frac{y'}{y}\frac{L}{l} = -\frac{l+\sigma}{f'}\frac{L}{l} = -\left(1+\frac{\sigma}{l}\right)\frac{L}{f'}$$

Speciální případ → y je v F → $P' = -\infty$ → $l \to \infty$ → $\Gamma_{MG} = -\frac{L}{f'} = \frac{L}{f}$

Mikroskop



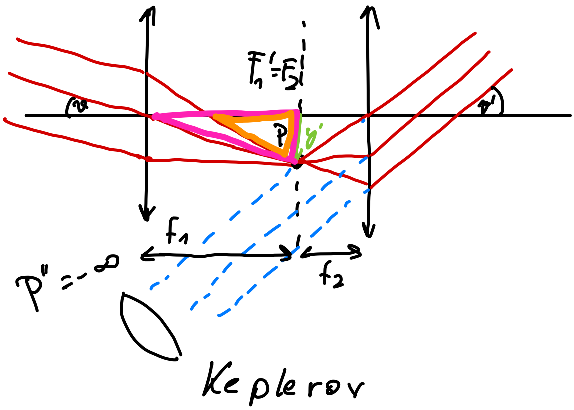
$\vartheta_L = -\frac{y}{L}$ $\quad$ $\vartheta' = \frac{y'}{f_2}$

$$\Gamma_{MIC} = \frac{\frac{y'}{f_2}}{-\frac{y}{L}} = -\underbrace{\frac{y'}{y}}_{M_T}\overbrace{\frac{L}{f_2}}^{\Gamma_{MG}(\infty)} = -\frac{\Delta}{f_1'}\frac{L}{f_2'}$$

M_T příčné zvětšení objektivu

$\frac{y'}{y} = \frac{\Delta}{f_1'}$

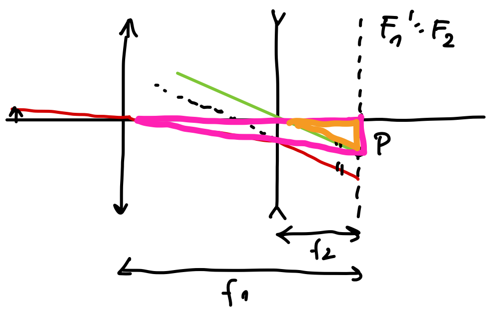
Dalekohlad



$$\vartheta = \frac{y'}{f_1'}$$

$$\vartheta' = \frac{y'}{f_2}$$

$$\Gamma = \frac{\vartheta'}{\vartheta} = \frac{\frac{y'}{f_2}}{\frac{y'}{f_1'}} = \frac{f_1'}{f_2}$$



$$\Gamma = \frac{f_1'}{f_2}$$

Fotoaparát

Anizotropné prostredí

izotropné

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

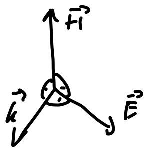
$\vec{P} \parallel \vec{E}$

$\vec{P} = \varepsilon_0 \chi \vec{E}$

$\nabla \cdot \vec{D} = 0$

$\vec{k} \cdot \vec{D} = 0$

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

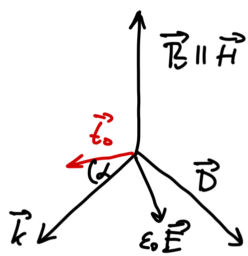


anizotropné

$\vec{P} = \varepsilon_0 \overleftrightarrow{\chi} \vec{E} \qquad \overleftrightarrow{\varepsilon_r} = 1 + \overleftrightarrow{\chi}$

↳ $\overleftrightarrow{\chi}$ diagonalizujeme: $\overleftrightarrow{\chi} = \begin{pmatrix} \chi_x & & \\ & \chi_y & \\ & & \chi_z \end{pmatrix}$

$$\overleftrightarrow{\varepsilon} = \begin{pmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{pmatrix} = \begin{pmatrix} n_1^2 & & \\ & n_2^2 & \\ & & n_3^2 \end{pmatrix}$$



$\vec{S} = \vec{E} \times \vec{H} = \vec{s}_0$

Fresnelova rovnice

$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$\vec{k} \times \vec{E} = \omega \vec{B} = \omega \mu_0 \vec{H}$

$\vec{H} = \dfrac{1}{\omega \mu_0} (\vec{k} \times \vec{E})$

$\nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}$

$\vec{k} \times \vec{H} = -\omega \vec{D} = -\omega \varepsilon_0 \overleftrightarrow{\varepsilon} \vec{E}$

$(\vec{k} \times \vec{k} \times \vec{E}) = -\omega^2 \varepsilon_0 \mu_0 \overleftrightarrow{\varepsilon} \vec{E}$

$\vec{k}(\vec{k} \cdot \vec{E}) - \vec{E}(\vec{k} \cdot \vec{k}) = -\dfrac{\omega^2}{c^2} \overleftrightarrow{\varepsilon} \vec{E}$

$\cancel{\dfrac{\omega^2}{c^2}} n^2 \vec{s}_0 (\vec{s}_0 \cdot \vec{E}) - \cancel{\dfrac{\omega^2}{c^2}} n^2 \vec{E} = -\cancel{\dfrac{\omega^2}{c^2}} \overleftrightarrow{\varepsilon} \vec{E}$

$n^2 \vec{s}_0 (\vec{s}_0 \cdot \vec{E}) - n^2 E_x = n_1^2 E_x$

$$\left.\begin{aligned} (n^2 - n_1^2) E_x &= n^2 s_{0x} (\vec{s}_0 \cdot \vec{E}) \\ (n^2 - n_2^2) E_y &= n^2 s_{0y} (\vec{s}_0 \cdot \vec{E}) \\ (n^2 - n_3^2) E_z &= n^2 s_{0z} (\vec{s}_0 \cdot \vec{E}) \end{aligned}\right\} \quad n^2 \text{ je neznáma}$$

Fresnelovy rovnice

$\overleftrightarrow{\varepsilon} = \begin{pmatrix} n_1^2 & & \\ & n_2^2 & \\ & & n_3^2 \end{pmatrix}$

$$
\left.\begin{aligned}
S_{0x} E_x &= \frac{n^2}{n^2-n_1^2}\, s_{0x}^2 (\vec{s}_0 \cdot \vec{E}) \\
S_{0y} E_y &= \frac{n^2}{n^2-n_2^2}\, s_{0y}^2 (\vec{s}_0 \cdot \vec{E}) \\
S_{0z} E_z &= \frac{n^2}{n^2-n_3^2}\, s_{0z}^2 (\vec{s}_0 \cdot \vec{E})
\end{aligned}\right\} + \;\rightarrow\; \cancel{\vec{s}_0 \cdot \vec{E}} = n^2 \left(\frac{S_{0x}^2}{n^2-n_1^2} + \frac{S_{0y}^2}{n^2-n_2^2} + \frac{S_{0z}^2}{n^2-n_3^2} \right) \cancel{(\vec{s}_0 \cdot \vec{E})}
$$

$$
\Rightarrow \boxed{\frac{1}{n^2} = \frac{S_{0x}^2}{n^2-n_1^2} + \frac{S_{0y}^2}{n^2-n_2^2} + \frac{S_{0z}^2}{n^2-n_3^2}}
$$

→ pro $n \neq n_i$ $\quad i=1,2,3$

Fresnelova rovnice

$a = (n_1^2 - n^2) \qquad b = (n_3^2 - n^2)$

$n_1 = n_2 \Leftrightarrow$ jednoosové materiály

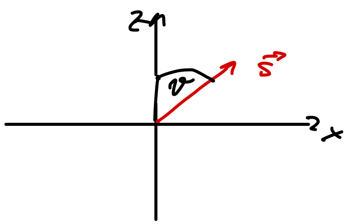
$$\downarrow$$

$$a\left[ab + n^2\left(a s_{0z}^2 + b s_{0y}^2 + b s_{0x}^2\right)\right] = 0$$

$a = 0 \Rightarrow n = n_1$ → řádná vlna, značíme o

řešení je mimořádná vlna e

Řešení v rovině x-z



$\vec{s}_0 = (s_{0x}, 0, s_{0y}) = (\sin\vartheta, 0, \cos\vartheta)$

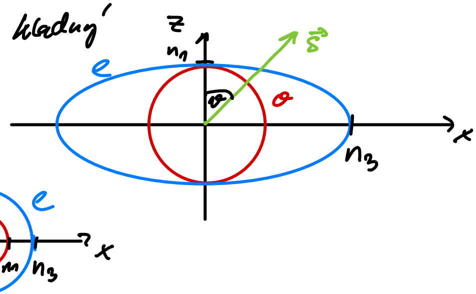
↓ dosazením do rovnice

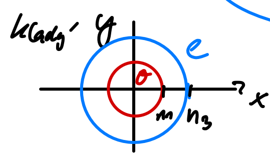
$$\frac{1}{n_e^2} = \frac{\sin^2\vartheta}{n_3^2} + \frac{\cos^2\vartheta}{n_1^2} =$$

$$\rightarrow\; 1 = \frac{\overbrace{n_e^2 \sin^2\vartheta}^{x_e}}{n_3^2} + \frac{\overbrace{n_e^2 \cos^2\vartheta}^{z_e}}{n_1^2}$$

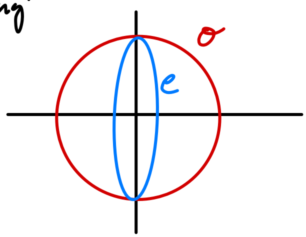
$n_3 > n_1 = n_o$ kladný smysl

$n_3 < n_1 = n$ záporný smysl





Záporný



$$(n^2-n_1^2)E_x = n^2 s_{0x}(\vec{s}_0\cdot\vec{E}) = n^2\sin\vartheta(E_x\sin\vartheta + E_z\sin\vartheta)$$
$$(n^2-n_1^2)E_y = n^2 s_{0y}(\vec{s}_0\cdot\vec{E}) = 0$$
$$(n^2-n_3^2)E_z = n^2 s_{0z}(\vec{s}_0\cdot\vec{E}) = n^2\cos\vartheta(E_x\sin\vartheta + E_z\cos\vartheta)$$

1) $n = n_1 = n_o$

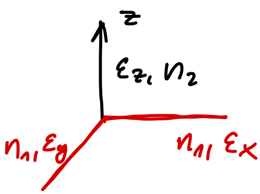
$\Rightarrow E_x\sin\vartheta + E_z\cos\vartheta = 0 \Rightarrow E_z = 0,\ E_x = 0 \Rightarrow \vec{E}_o = (0, E_y, 0)$

$\Rightarrow \vec{E}_o \perp$ na rovinu hlavního řezu

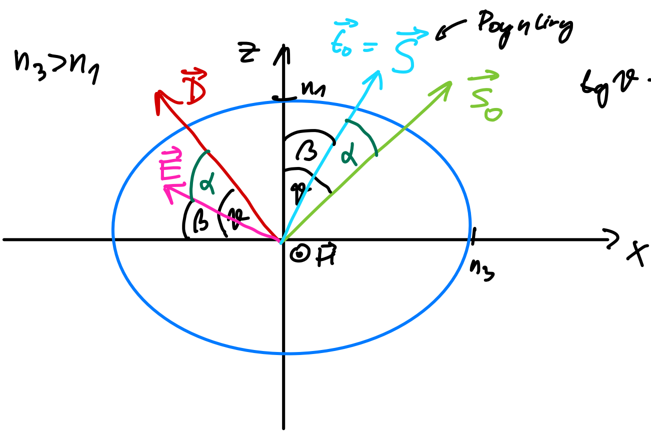
2) $(n_e^2 - n_1^2)E_y = 0 \Rightarrow E_y = 0,\ \vec{E}_e = (E_x, 0, E_z)$

↳ v rovině hlavního řezu

$\vec{E}_e \perp \vec{E}_o$



$n_3 > n_1$



$$\operatorname{tg}\vartheta = \frac{D_z}{D_x}$$

$$D_x = \varepsilon_0\varepsilon_x E_x = \varepsilon_0 n_1^2 E_x$$
$$D_z = \varepsilon_0\varepsilon_z E_z = \varepsilon_0 n_3^2 E_z$$

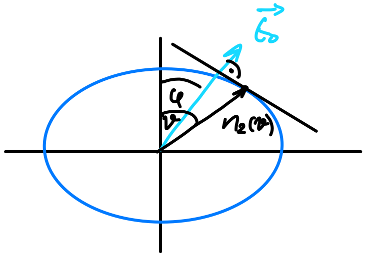
$$\Rightarrow \frac{D_x}{D_z} = \frac{n_3^2}{n_1^2}\frac{E_z}{E_x}$$

$$\operatorname{tg}\beta = \frac{E_z}{E_x}$$

$$\Rightarrow \operatorname{tg}\vartheta = \frac{n_3^2}{n_1^2}\operatorname{tg}\beta$$

$$\alpha = \vartheta - \beta$$

→ směr $\vec{t}_0$ je dán normálou k tečné ploše elipsy



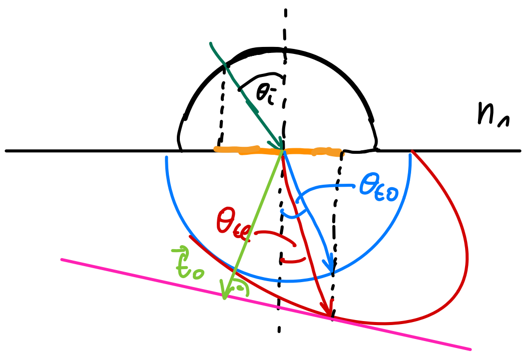
$$\frac{x_e^2}{n_3^2} + \frac{z_e^2}{n_1^2} - 1 = 0$$

$$\hat{N} = \frac{\left(\frac{\partial E}{\partial x}, \frac{\partial E}{\partial y}, \frac{\partial E}{\partial z}\right)}{\sqrt{\left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 + \left(\frac{\partial E}{\partial z}\right)^2}}$$

$$\rightarrow \vec{N} = \left(\frac{2x_e}{n_3^2}, 0, \frac{2z_e}{n_1^2}\right) = 2\left(\frac{n_e \sin\vartheta}{n_3^2}, 0, \frac{n_e \cos\vartheta}{n_1^2}\right)$$

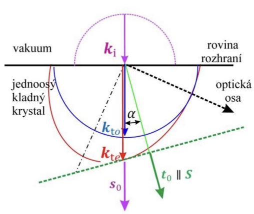
$$tg\,\varphi = \frac{\frac{\sin\vartheta}{n_2^2}}{\frac{\cos\vartheta}{n_1^2}} = \frac{\sin\vartheta}{\cos\vartheta}\frac{n_1^2}{n_2^2} = tg\,\vartheta \frac{n_1^2}{n_2^2} \quad \Rightarrow \varphi = \beta$$

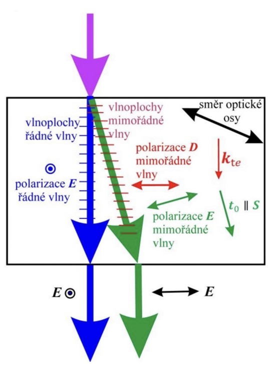
Lom na rozhraní s AI prostředí



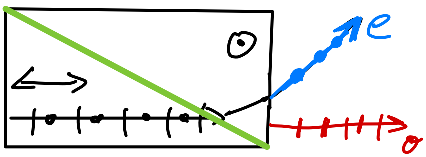
$$n_1 \sin\theta_i = n_2 \sin\theta_{to}$$
$$n_1 \sin\theta_i = n_2(\vartheta) \sin\theta_{te}$$

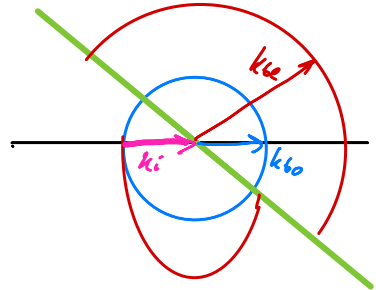
pro kolmý dopad:



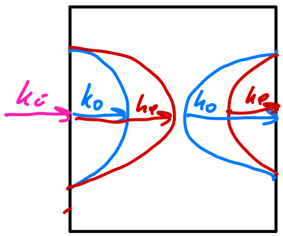


Rochonův pol hranol



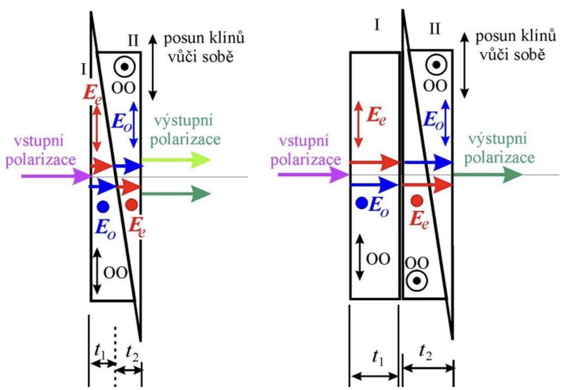


Fázová destička



$$\Delta\varphi = \varphi_e - \varphi_o = (k_e - k_o)d = \frac{\omega}{c}(n_3 - n_1)d$$

Kompenzátor



$$\varphi_1 = \frac{2\pi}{\lambda_0}(n_o t_1 + n_e t_2)$$

$$\varphi_2 = \frac{2\pi}{\lambda_0}(n_e t_1 + n_o t_2)$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = \frac{2\pi}{\lambda_0}(n_e - n_o)(t_2 - t_1)$$

Optická indikatrix

$$w_e = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}(E_x D_x + E_y D_y + E_z D_z) =$$

$$= \frac{1}{2}\left(\frac{D_x^2}{\varepsilon_0 n_1^2} + \frac{D_y^2}{\varepsilon_0 n_2^2} + \frac{D_z^2}{\varepsilon_0 n_3^2}\right)$$

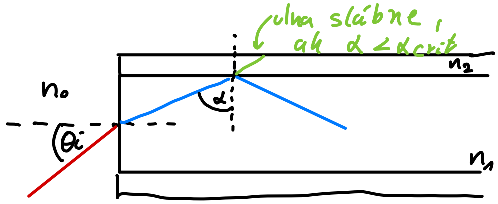
$$\underbrace{2\varepsilon_0 w_e}_{=1} = \frac{D_x^2}{n_1^2} + \frac{D_y^2}{n_2^2} + \frac{D_z^2}{n_3^2}$$

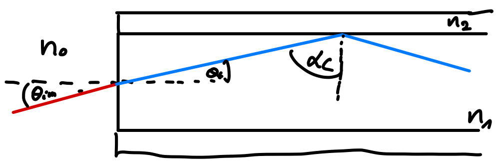
$$\Rightarrow \frac{D_x^2}{n_1^2} + \frac{D_y^2}{n_2^2} + \frac{D_z^2}{n_3^2} = 1$$

Vláknová optika

- vlnovody – založeny na jevu totálního odrazu

$n_2 > n_1$





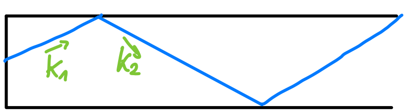
$$\theta_t + \alpha_c = \frac{\pi}{2}$$

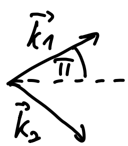
$$n_1 \sin \alpha_c = n_2 \qquad n_0 \sin \theta_{im} = n_1 \sin \theta_t = n_1 \cos \alpha_c$$

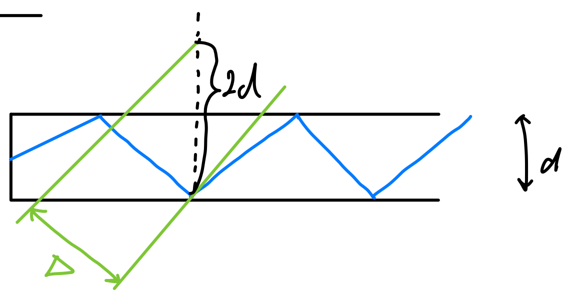
$$n_1^2 \sin^2 \alpha_c = n_2^2 \qquad n_0^2 \sin^2 \theta_{im} = n_1^2 \cos^2 \alpha_c$$

$$\Rightarrow n_0^2 \sin^2 \theta_{im} = n_1^2 \left(1 - \sin^2 \alpha_c\right) = n_1^2 \left(1 - \frac{n_2^2}{n_1^2}\right)$$

$$NA = n_0 \sin \theta_{im} = \sqrt{n_1^2 - n_2^2}$$ → numerická apertura





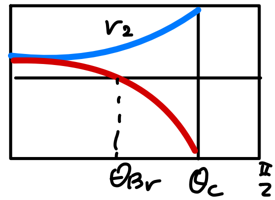


$$\cos\alpha = \frac{\Delta}{2d}$$

$$\Delta = 2d \cos\alpha$$

$\Delta\varphi = 2k_0 n_1 2d\cos\alpha + \Delta\varphi_{odr} = 2m\pi$

$k_0 = \frac{2\pi}{\lambda_0}$

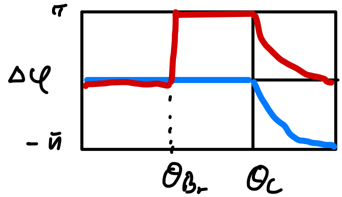


Odhad počtu módů $\Delta\varphi_{odr} \approx 0$

$\alpha = \alpha_c$

$$\cancel{2}k_0 d n_1 \cos\alpha_c = \cancel{2} m\pi$$

$$\rightarrow \frac{2\cancel{\pi}}{\lambda_0} d \underbrace{n_1\cos\alpha_c}_{NA} = m\cancel{\pi}$$



$m = 0, 1, \dots, M$

MAX $\left[\frac{2d}{\lambda_0} NA\right]_{\text{celá část}} = M$ → počet módů, tj. se mohou šířit je $M+1$

Útlum $10\log_{10}\frac{P_1}{P_2}$ [dB] P_1 ... výkon na vstupu

P_2 ... výkon na výstupu

Nelineárne optické jevy

$$\vec{P} = \varepsilon_0 \chi \vec{E} \qquad \vec{P} = \varepsilon_0 \overleftrightarrow{\chi} \vec{E} \qquad \text{symbolicky: } P = \varepsilon_0 \chi \vec{E} + \underbrace{\varepsilon_0 \chi^{(2)} E^2}_{P^{(2)}} + \underbrace{\varepsilon_0 \chi^{(3)} E^3}_{P^{(3)}}$$

↳ izotropní lineární ↳ anizotropní lineární ↳ nelineárne

$\chi^{(2)}$ → tenzor 3. řádu; $\chi^{(3)}$ → tenzor 4. řádu

$$P_i = \varepsilon_0 \chi_{ij}(\omega) E^j(\omega) + \varepsilon_0 \chi_{ijk}(\omega_i, \omega_j, \omega_k) E^j(\omega_j) E^k(\omega_k) + \varepsilon_0 \chi_{ijkl}(\omega_i, \omega_j, \omega_k, \omega_l) E^j(\omega_j) E^k(\omega_k) E^l(\omega_l)$$

2. řádu →

Časová závislost

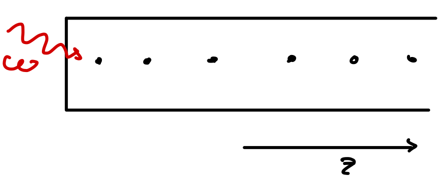
$$E = E_0 \cos\omega t = E_0 \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$$

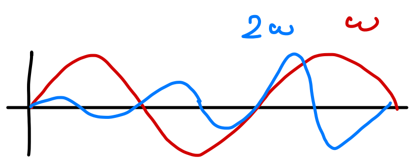
$$P_i^{(2)} = \varepsilon_0 \chi_{ijk} E^j E^k$$

→ po zmene $\left\{\begin{array}{l} E^j \to -E^j \\ E^k \to -E^k \\ P_i \to -P_i \end{array}\right\}$ dostaneme, že $P_i = 0$, teda v stredovo symet. látkach sa nevyskytují 2. řádu

$$P^{(2)} = \varepsilon_0 \chi^{(2)} \frac{E_0^2}{2}\left(e^{i2\omega t} + e^{-i2\omega t} + 2\right) = \underbrace{\varepsilon_0 \chi^{(2)} \frac{E_0^2}{2}}_{\text{optické usměrnení (statická polarizace)}} + \varepsilon_0 \chi^{(2)} \frac{E_0^2}{2} \underbrace{\cos(2\omega t)}_{\text{generace 2. harmonické frekvence}}$$

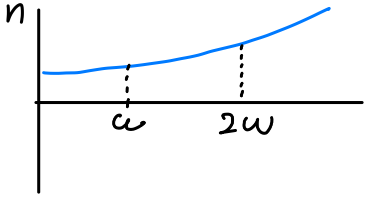
Podmínka stažování





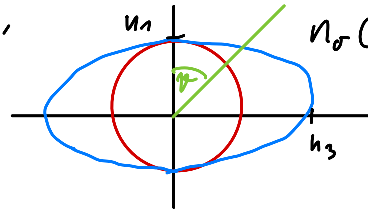
$$\omega \ldots k_1 = \frac{\omega}{c} n(\omega)$$

$$2\omega \ldots k_2 = \frac{2\omega}{c} n(\omega)$$



Podmínka sfázovaní $\quad 2k_1 = k_2 \rightarrow 2k_1 - k_2 = 0 \rightarrow \frac{2\omega}{c} n(\omega) = \frac{2\omega}{2} n(2\omega)$

↳ anizotropní prostředí



$n_o(2\omega) = n_e(\omega)$

3. řádu

$$P^{(3)} = \varepsilon_0 \chi^{(3)} E^3 = \varepsilon_0 \chi^{(3)} \frac{E_0^3}{8} \left(e^{i\omega t} + e^{-i\omega t}\right) =$$

$$= \frac{\varepsilon_0 \chi^{(3)} E_0^3}{8} \left(e^{i3\omega t} + e^{-i3\omega t} + 3e^{i\omega t} + 3e^{-i\omega t}\right) =$$

$$= \varepsilon_0 \chi^{(3)} \frac{E_0^3}{4} \underbrace{\cos(3\omega t)}_{\text{generace 3. harmonické}} + \underbrace{\frac{3}{4} \varepsilon_0 \chi^{(3)} E_0^3 \cos \omega t}_{\text{závislost indexu lomu na intenzitě } n(I)}$$

$$P = \underset{\text{lin.}}{P_L} + \underset{\text{nelin.}}{P_{NL}} = \varepsilon_0 \chi_L E + \varepsilon_0 \underbrace{\left(\frac{3}{4}\chi^{(3)} E^2\right)}_{\chi_{NL}} E =$$

$$= \varepsilon_0 (\chi_L + \chi_{NL}) E = \varepsilon_0 \chi E$$

$$I = \frac{1}{2} \varepsilon_0 n_L c E_0^2 \rightarrow E_0^2 = \frac{2I}{\varepsilon_0 n_L c} \rightarrow \chi_{NL} = \frac{3}{4} \frac{\chi^{(3)} 2I}{\varepsilon_0 n_L c}$$

index lomu $\rightarrow$ $n(\omega) = n_L(\omega) + n_{NL}(\omega)$

$$n(\omega) = \sqrt{\varepsilon_r(\omega)} = \sqrt{1 + \chi_L + \chi_{NL}} = \sqrt{1+\chi_L} \sqrt{1 + \frac{\chi_{NL}}{1+\chi_L}}$$

↳ $\varepsilon_r(\omega) = 1 + \chi_L + \chi_{NL}$

$$n(\omega) = \underbrace{\sqrt{1+\chi_L}}_{n_L}\left(1 + \frac{\chi_{NL}}{2\underbrace{(1+\chi_L)}_{n_L^2}}\right) = n_L + \frac{\chi_{NL}}{2n_L}$$

Taylor

$$\Rightarrow n(\omega) = n_L + \frac{3}{4}\frac{\chi^{(3)} I}{\varepsilon_0 n_L^2 c} = n_L + \frac{1}{2} n_2 I$$

$$n_2 = \frac{3\chi^{(3)}}{2n_L^2 \varepsilon_0 c}$$

Interakce světla s látkou

Klasický popis

- homogenní, izotropní látka
- kolmý dopad světla na látku

→ absorbce

$$I = I_0 e^{-\alpha z}$$

α .... extinkční koeficient

↳ při absorbci = absorbční koef.

$$\frac{dI}{dz} = -\alpha I$$

$$E = E_0 e^{-k_I z} e^{i(k_R z - \omega t)} = E_0 e^{i(\tilde{k} z - \omega t)}$$

$\tilde{k} = k_R + i k_I$

↑ komplexní vlnový vektor

$$k_R = \frac{\omega}{c} n \qquad k_I = \frac{\omega}{c} \kappa$$

$$\tilde{k} = |\tilde{k}| e^{i\varphi}$$

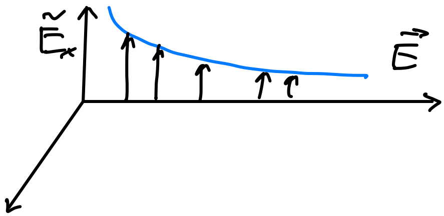
$\hat{N} = \frac{\omega}{c}(n + i\kappa)$ → komplexní index lomu

$$\vec{P} = \varepsilon_0 \tilde{\chi} \vec{E}$$
$$\vec{D} = \varepsilon_0 \tilde{\varepsilon}_r \vec{E}$$

$$\tilde{\chi} = \chi_R + i\chi_I$$
$$\tilde{\varepsilon}_r = \varepsilon_R + i\varepsilon_I$$

$\tilde{\sigma} = \sigma_R + i\sigma_I$ → cplx vodivost

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{B} = \mu_0 \vec{H}$$



$$\left(0, \frac{\partial E_x}{\partial z}, 0\right) = -\partial_t \vec{B}$$

$$\parallel$$

$$i E_0 \tilde{k}\, e^{-i(\tilde{k} z - \omega t)} = \partial_t \vec{B}$$

$$B_y = \frac{i E_0 \tilde{k}}{i\omega} = E_0 \frac{\tilde{k}}{\omega} e^{-i(\tilde{k} z - \omega t)}$$

$$\tilde{B}_y = \tilde{E}_x \frac{\tilde{k}}{\omega}$$

$$\Rightarrow \tilde{B}_y = \frac{|\tilde{k}|}{\omega} e^{i\varphi_N} \tilde{E}_x$$

fázový posun voči E_x