Implicitní funkce z cvičení 5

Příklad č. 4:
$M := \left\{[x,y] \in \mathbb{R}^2; x \sin y = y\right\}, a = \left[\frac{\pi}{2}, \frac{\pi}{2}\right]$
$F(x,y) = x \sin y - y$
$F(a) = 0$
$\frac{\partial F}{\partial y}(a) = x \cos y - 1 = -1 \neq 0, F \in C^1$
$\Rightarrow$splněny předpoklady věty o implicitní funkci
$\Rightarrow \varphi(x)$ v okolí $\frac{\pi}{2}$ existuje
$\sin y + x (\cos y) y' = y' \Rightarrow y'\left(\frac{\pi}{2}\right) = 1$

Příklad č. 5:
$M := \left\{[x,y] \in \mathbb{R}^2; e^x y + \cos\left(yx^2\right) = x\right\}, a = [1,0]$
$F(x,y) = e^x y + \cos\left(yx^2\right) - x$
$F(a) = 0$
$\frac{\partial F}{\partial y}(a) = e^x - \sin\left(yx^2\right) x^2 = e \neq 0, F \in C^1$
$\Rightarrow$splněny předpoklady věty o implicitní funkci
$\Rightarrow \varphi(x)$ v okolí 1 existuje
$e^x y + e^x y' - \sin\left(yx^2\right)\left(y'x^2 + 2xy\right) = 1 \Rightarrow y'(1) = \frac{1}{e}$

Příklad č. 6:
$M := \left\{[x,y,z] \in \mathbb{R}^3; 2x - z^2 - xy + y = 0\right\}, a = [2,0,2]$
$F(x,y,z) = 2x - z^2 - xy + y$
$F(a) = 0$
$\frac{\partial F}{\partial z}(a) = -2z = -4 \neq 0, F \in C^1$
$\Rightarrow$splněny předpoklady věty o implicitní funkci
$\Rightarrow \varphi(x,y)$ v okolí $[2,0]$ existuje
$\frac{\partial F}{\partial x}(a) = 2 - y = 2$
$\frac{\partial F}{\partial y}(a) = -x + 1 = -1$
$D_\varphi(a)(h) = \frac{\partial \varphi}{\partial x}(a) h_1 + \frac{\partial \varphi}{\partial y}(a) h_2 = -\frac{\frac{\partial F}{\partial x}(a)}{\frac{\partial F}{\partial z}(a)} h_1 - \frac{\frac{\partial F}{\partial y}(a)}{\frac{\partial F}{\partial z}(a)} h_2 = -\frac{1}{2} h_1 + \frac{1}{4} h_2$

Příklad č. 7:
$M := \left\{[x,y,z] \in \mathbb{R}^3; x + \sin y + e^{xz} = y\right\}, a = [-1,0,0]$
$F(x,y,z) = x + \sin y + e^{xz} - y$
$F(a) = 0$
$\frac{\partial F}{\partial z}(a) = xe^{xz} = -1 \neq 0, F \in C^1$
$\Rightarrow$splněny předpoklady věty o implicitní funkci
$\Rightarrow \varphi(x,y)$ v okolí $[-1,0]$ existuje
$\frac{\partial F}{\partial x}(a) = 1 + ze^{xz} = 1$
$\frac{\partial F}{\partial y}(a) = \cos y - 1 = 0$
$D_\varphi(a)(h) = \frac{\partial \varphi}{\partial x}(a) h_1 + \frac{\partial \varphi}{\partial y}(a) h_2 = -\frac{\frac{\partial F}{\partial x}(a)}{\frac{\partial F}{\partial z}(a)} h_1 - \frac{\frac{\partial F}{\partial y}(a)}{\frac{\partial F}{\partial z}(a)} h_2 = -h_1$

Příklad č. 8:
$F_1 = x^2 + u - ux - v$
$F_2 = x^3 + y - \sin(xv) - u$
$F_3 = 2x - z - uv - u$
$a = [0,0]$
$F_1(a) = 0, F_2(a) = 0, F_3(a) = 0$
$\frac{\partial F_1}{\partial x}(a) = 1 - x = 1, \frac{\partial F_1}{\partial y}(a) = -1, \frac{\partial F_1}{\partial z}(a) = 0$
$\frac{\partial F_2}{\partial x}(a) = -1, \frac{\partial F_2}{\partial y}(a) = -x \cos(xv) = 0, \frac{\partial F_2}{\partial z}(a) = 0$
$\frac{\partial F_3}{\partial x}(a) = -v - 1 = -1, \frac{\partial F_3}{\partial y}(a) = -u = 0, \frac{\partial F_3}{\partial z}(a) = -1$
$F_1, F_2, F_2 \in C^1$

$|F'_u(a), F'_v(a), F'_z(a)| = \begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 1 \end{vmatrix} = -1 \neq 0$
$\Rightarrow$splněny předpoklady věty o implicitních funkcích a tedy $u(x,y)$, $v(x,y)$ a $z(x,y)$ jsou v okolí bodu $[0,0]$ definovány a jsou diferencovatelné, tedy mají derivaci a jsou tak hladké
$\frac{\partial F_1}{\partial x}(a) = -u = 0$, $\frac{\partial F_2}{\partial x}(a) = 3x^2 - x\cos(xv) = 0$, $\frac{\partial F_3}{\partial x}(a) = 2$
$\frac{\partial F_1}{\partial y}(a) = 0$, $\frac{\partial F_2}{\partial y}(a) = 1$, $\frac{\partial F_3}{\partial y}(a) = 0$
$\frac{\partial z}{\partial x}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 2 \end{vmatrix}}{-1} = 2$, $\frac{\partial z}{\partial y}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix}}{-1} = -1$
$D_z(a)(h) = 2h_1 - h_2$

Příklad č. 11:
$F_1 = x - u^3 + uv$
$F_2 = y - 3uv - 2v^2$
$a = [-1, 2, 1, 2]$
$F_1(a) = 0, F_2(a) = 0$
$\frac{\partial F_1}{\partial u}(a) = -3u^2 + v = -1$, $\frac{\partial F_1}{\partial v}(a) = u = 1$
$\frac{\partial F_2}{\partial v}(a) = -3v = -6$, $\frac{\partial F_2}{\partial v}(a) = -3u - 4v = -5$
$F_1, F_2 \in C^1$
$|F'_u(a), F'_v(a)| = \begin{vmatrix} -1 & -1 \\ -6 & -5 \end{vmatrix} = 1 \neq 0$
$\Rightarrow$splněny předpoklady věty o implicitních funkcích a tedy $u(x,y)$ a $v(x,y)$ jsou v okolí bodu $[1,2]$ definovány a jsou diferencovatelné
$\frac{\partial F_1}{\partial x}(a) = 1$, $\frac{\partial F_1}{\partial y}(a) = 0$, $\frac{\partial F_2}{\partial x}(a) = 0$, $\frac{\partial F_2}{\partial y}(a) = 1$
$\frac{\partial u}{\partial x}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix}}{-1} = 5$, $\frac{\partial u}{\partial y}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} 0 & -1 \\ 1 & -5 \end{vmatrix}}{-1} = -1$
$D_u(h)(a) = 5h_1 - h_2$
$\frac{\partial v}{\partial x}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} -1 & 1 \\ -6 & 0 \end{vmatrix}}{-1} = -6$, $\frac{\partial v}{\partial y}(a) = \frac{|A'|}{|F'|} = \frac{\begin{vmatrix} -1 & 0 \\ -6 & 1 \end{vmatrix}}{-1} = 1$
$D_v(h)(a) = -6h_1 + h_2$