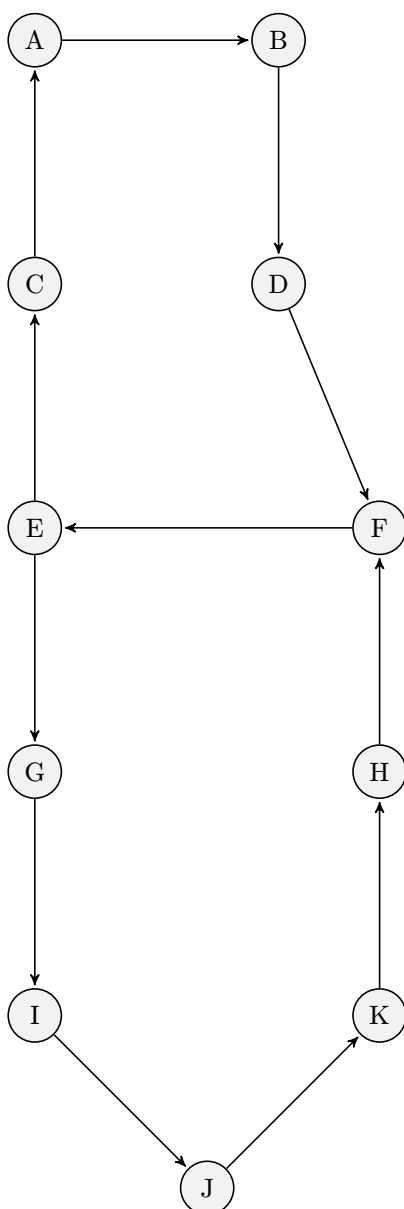# PAST 2 – 3.2.2026

Zkouška trvala 150 minut, 1 A4 poznámek. Až na úlohu 3 nebyla ve zkoušce čísla, všude chtěli vše vyjádřit vzorečkem. Snažil jsem se napsat zadání co nejpřesněji, ale je to pouze zkrácený přepis. Názvy úloh jsou ode mě.

Kdyby něco nebylo jasné, napiš mail na `ouolim@eideo.cz` a dovysvětlím. Godspeed.

Q1: Markov chain

Note: If there are multiple edges leaving a vertex, their probabilities are uniform (equal).

1) List states accessible from F.
2) List all communicating classes.

3) Decide if a stationary distribution exists. If there is at least one, write it down. If there is more than one, can you characterize them?
4) Let $X_0 = E$, compute $P(X_{10^{20}} = E)$.

Q2: Balls n bins

You are a farmer buying ingredients at a market. There are n ingredients, and at each step, you roll an n-sided die to decide what you are going to buy.
1) Compute the expected amount of rolls until half of the ingredients are bought. (Can be expressed as a sum of n simple terms).
2) Compute the expected amount of rolls until each ingredient is bought twice. You don't need to actually compute it, rather explain how you would compute it in polynomial time.

Q3: Bayes

There are 3 factories A, B and C. Factory A made $\frac{1}{3}$ of all cars in the world, factory B made $\frac{2}{6}$ cars and factory C the rest. Factory A produces a defect on wheels with probability 0.01%, the other two with probability 0.005%.
Also, factories A and B produce cars with a pedal defect with probability 0.001%, factory C with probability 0.02%.
You experienced both defects (on the same car). What factory do you believe the car came from?

Q4: Balls n bins 2.0

A function maps k keys into b buckets. Let X_i be the number of keys in bucket i. Compute:
1) $E[emptybuckets]$
2) Expected excess defined as: $\sum_{i=1}^{b} \max(0, X_i - 1)$
3) Expected number of buckets with ≥ 2 items (collisions).
4) Assume $k = b$, compute the maximum bucket load.
5) Compute the upper bound for b so at least one bucket has load $\geq \frac{2k}{b}$.

Q5: MGF

1) Define moment generating function (MGF).
2) X is a random variable generated by a 6-sided die. Determine $MGF(X)$.
3) Y is a random variable equal to the sum of rolling a 6-sided die four times. Determine $MGF(Y)$.

Q6: Gadgets

A factory makes gadgets, each gadget is faulty with probability p. Compute:
1) Expected number of faulty gadgets in the first n items.
2) P(first n gadgets are ok).
3) Let $T =$index of the first defective gadget. Compute $E[T]$.
4) The factory stops after two faulty gadgets in a row. Let S be the index of the gadget on which the factory stopped. Compute $E[S]$.