1)
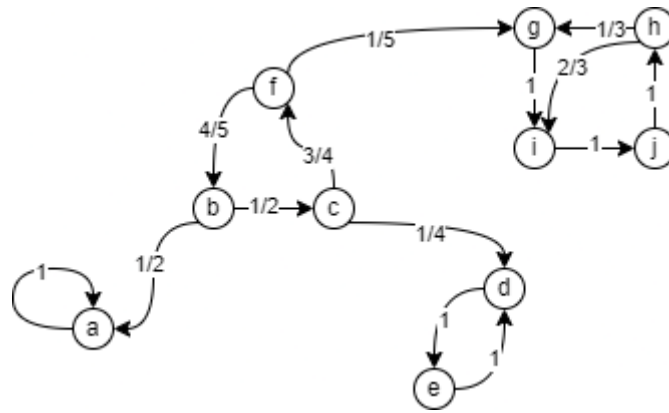

a) Find communicating components

b) State which states are transient, recurrent, periodic (+its period). Explain why.

c) $P(\text{reaching h anytime in the future}|X_0 = b)$

d) $P(X_{3000} = h|X_0 = b)$

2)

a) Define $\mathbb{E}(X|Y)$

b) Number of employees assigned to project is random variable $N = Geom(1/3)$. Hours spend on the project is $H = Exp(1/(2n + 3))$ where $n \sim N$. Compute $\mathbb{E}(H|N)$

c) Use Law of iterated expectation to find $\mathbb{E}(H)$

d) $Var(H) =?$

3)

Prior A: Uniform(0,1)

Prior B: $f_B(p) = cp^2 \ (p \in (0, 1), \ \text{suitable c}, \ f_B(p) = 0 \text{ outside } [0, 1])$

We saw 3 successes out of 7 observations.

a) Find posterior from prior A. Find MAP estimate.

b) How does the posterior from prior B differ from previous question?

c) What other estimate do you know? (You don't have to calculate, just the integral that you can directly put into computer is enough)

4)

a) Explain Poisson approximation for Balls and Bins (proof not needed)

b) For $3n$ balls and n bins, show using the Poisson approximation, that for n going to infinity, the probability that there exists a bin with exactly 2 balls approaches 1.

5)

a) Prove Central Limit Theorem using Moment Generating Function (proof for MGF properties is not needed)

6)

a) Chapman-Kolmogorov formula (state it + proof)